12

Positive Displacement Machines

The function of a compressor is to take a definite quantity of fluid (usually a gas, and most often air) and deliver it at a required pressure. The most efficient machine is one which will accomplish this with the minimum input of mechanical work. Both reciprocating and rotary positive displacement machines are used for a variety of purposes. On the basis of performance a general distinction can be made between the two types by defining the reciprocating type as having the characteristics of a low mass rate of flow and high-pressure ratios, and the rotary type as having a high mass rate of flow and low-pressure ratios. The pressure range of atmospheric to about 9 bar is common to both types.

Some rotary machines are suitable only for low-pressure ratio work, and are applied to the scavenging and supercharging of engines, and the various applications of exhausting and vacuum pumping. For pressures above 9 bar the vane-type rotary machine can be used to supply boost pressures, but for sustained high-pressure work up to 500 bar and above, for special purposes, the reciprocating type is used.

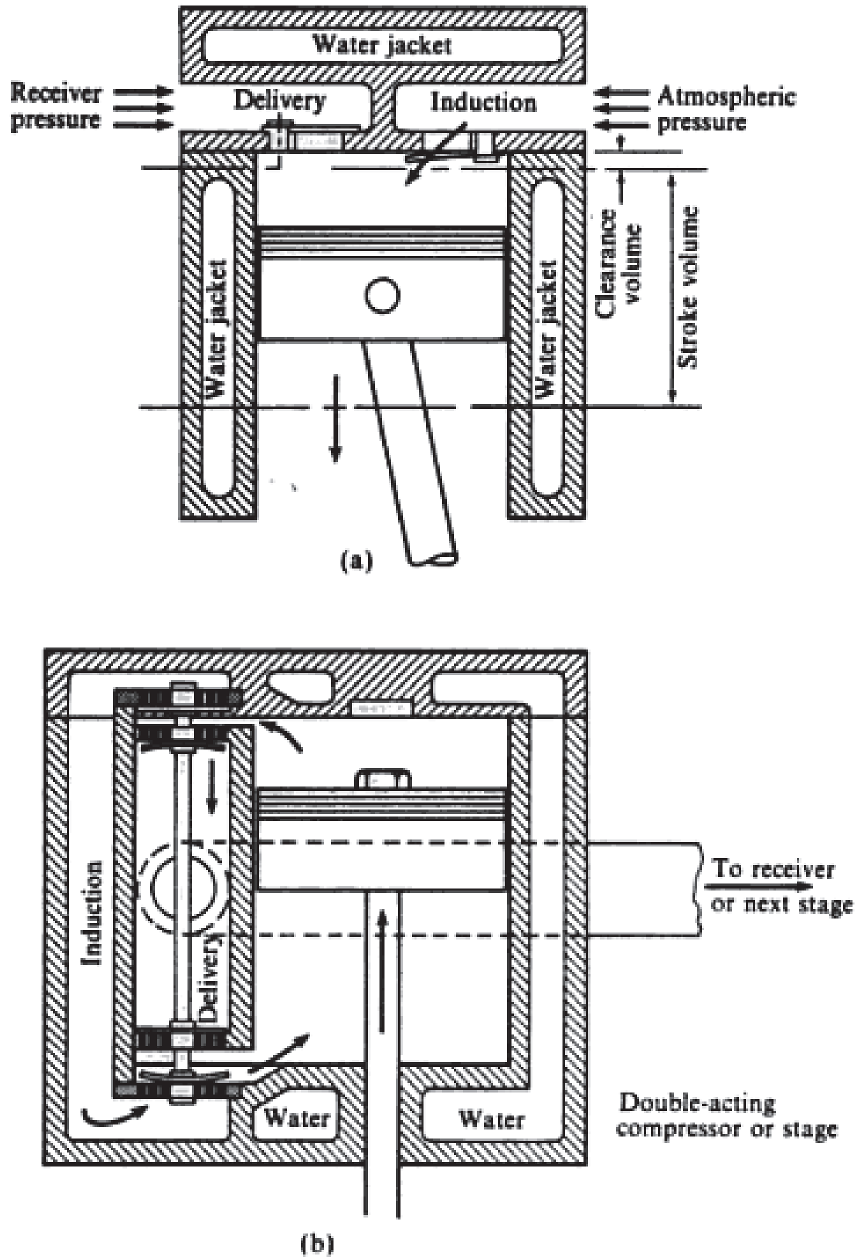
Both basic types exist in many different forms each having its own characteristics. They may be single or multistage, and have either air or water cooling. The reciprocating machine is pulsating in action which limits the rate at which fluid can be delivered, but the rotary machine is continuous in action and does not have this disadvantage. The rotary machines are smaller in size for a given flow, lighter in weight and mechanically simpler than their reciprocating counterparts. The treatment and scope of the following sections is fundamental and is not exhaustive. Many compressors are designed to overcome the deficiencies of the basic machines and to satisfy special requirements. For descriptions of these machines the excellent literature supplied by the manufacturers concerned should be consulted.

For a compressor which operates in a cyclic or pulsating manner, such as a reciprocating compressor, the properties at inlet and outlet are the average values taken over the cycle. Alternatively the boundary of the control volume is chosen such that states 1 and 2 are constant with time, the positions selected being remote from the pulsating disturbance.

12.1 Reciprocating compressors

Typical reciprocating compressor cylinder arrangements are shown in Fig. 12.1(a) and (b). The mechanism involved is the basic piston, connecting-rod,

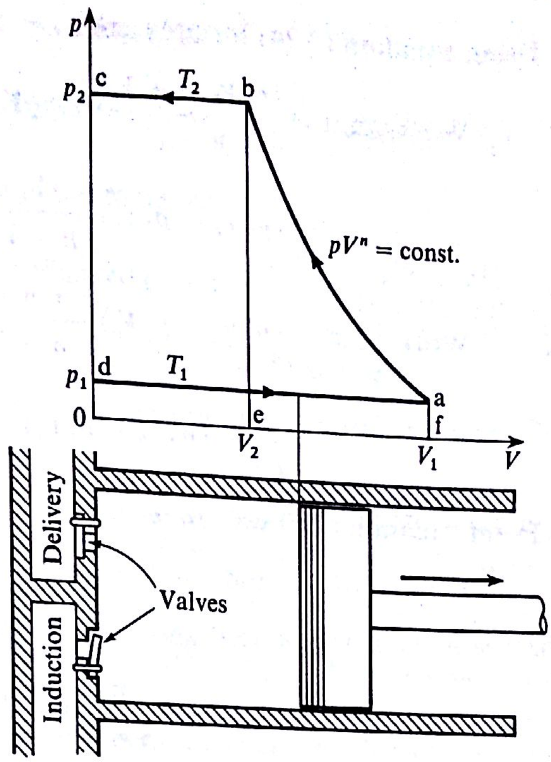
Fig. 12.1
Single-acting (a) and double-acting (b) reciprocating air compressors



crank, and cylinder arrangement. Initially the clearance volume in the cylinder will be considered negligible. Also the working fluid will be assumed to be a perfect gas. The cycle takes one revolution of the crankshaft for completion and the basic indicator diagram is shown in Fig. 12.2.

The valves employed in most air compressors are designed to give automatic action. They are of the spring-loaded type operated by a small difference in pressure across them, the light spring pressure giving a rapid closing action.

Fig. 12.2 Pressure–volume diagram for a reciprocating compressor with clearance neglected



The lift of the valve to give the required airflow should be as small as possible and should operate without shock.

In Fig. 12.2 the line d–a represents the induction stroke. The mass in the cylinder increases from zero at d to that required to fill the cylinder at a. In the ideal case the temperature is constant at T_1 for this process and there is no heat exchange with the surroundings. Induction commences when the pressure difference across the valve is sufficient to open it. Line abc represents the compression and delivery stroke. As the piston begins its return stroke the pressure in the cylinder rises and closes the inlet valve. The pressure rise continues with the returning piston as shown by line ab until the pressure is reached at which the delivery valve opens (a value decided by the valve and the pressure in the receiver). The delivery takes place as shown by the line bc, which is a process at constant temperature T_2, constant pressure p_2, zero heat exchange, and decreasing mass. At the end of this stroke the cycle is repeated. The value of the delivery temperature T_2 depends upon the law of compression between a and b, which in turn depends upon the heat exchange with the surroundings during this process. It may be assumed that the general form of compression is the reversible polytropic (i.e. pV^n = constant).

The net work done in the cycle is given by the area of the p–V diagram and is the work done on the gas.

Indicated work done on the gas per cycle

= area abcd

= area abef + area bc0e – area ad0f

Using equation (3.24) for area abef,

$$\text{Work input} = \frac{(p_2V_b - p_1V_a)}{n-1} + p_2V_b - p_1V_a$$

$$= (p_2V_b - p_1V_a)\left(\frac{1}{n-1} + 1\right)$$

i.e.
$$\text{Work input} = (p_2V_b - p_1V_a)\frac{1+n-1}{n-1}$$

$$= \frac{n}{n-1}(p_2V_b - p_1V_a) \qquad (12.1)$$

From equation (2.6) we can write

$$p_1V_a = mRT_1 \quad \text{and} \quad p_2V_b = mRT_2$$

where m is the mass induced and delivered per cycle. Then

$$\text{Work input per cycle} = \frac{n}{n-1}mR(T_2 - T_1) \qquad (12.2)$$

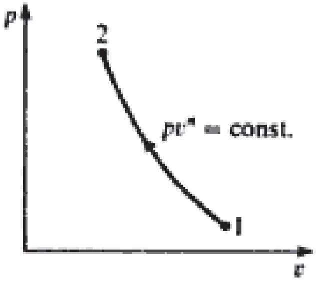


Fig. 12.3 Compression process on a p–v diagram

Work done on the air per unit time is equal to the work done per cycle times the number of cycles per unit time. The rate of mass flow is more often used than the mass per cycle; if the rate of mass flow is given the symbol $\dot{m}$, and replaces m in equation (12.2), then the equation gives the rate at which work is done on the air, or the indicated power.

The working fluid changes state between a and b in Fig. 12.2, from p_1 and T_1 to p_2 and T_2, the change being shown in Fig. 12.3, which is a diagram of properties (i.e. p against v).

The delivery temperature is given by the equation (3.29),

i.e.
$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

Example 12.1

A single-stage reciprocating compressor takes 1 m^3 of air per minute at 1.013 bar and 15 °C and delivers it at 7 bar. Assuming that the law of compression is $pV^{1.35}$ = constant, and that clearance is negligible, calculate the indicated power.

Solution

$$\text{Mass delivered per min, } \dot{m} = \frac{p_1V_1}{RT_1}$$

$$= \frac{1.013 \times 1 \times 10^5}{287 \times 288} = 1.226 \text{ kg/min}$$

where $T_1 = 15 + 273 = 288$ K.

$$\text{Delivery temp., } T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 288\left(\frac{7}{1.013}\right)^{(1.35-1)/1.35}$$

$$= 475.4 \text{ K}$$

From equation (12.2)

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}R(T_2 - T_1)$$

where $\dot{m}$ is the mass flow rate,

i.e. $$\text{Indicated power} = \frac{1.35 \times 1.226 \times 287 \times (475.4 - 288)}{10^3 \times (1.35 - 1) \times 60}$$

$$= 4.238 \text{ kW}$$

The actual power input to the compressor is larger than the indicated power, due to the work necessary to overcome the losses due to friction, etc.

i.e. $$\text{Shaft power} = \text{indicated power} + \text{friction power} \qquad (12.3)$$

The mechanical efficiency of the machine is given by

$$\text{Compressor mechanical efficiency} = \frac{\text{indicated power}}{\text{shaft power}} \qquad (12.4)$$

To determine the power input required the efficiency of the driving motor must be taken into account, in addition to the mechanical efficiency. Then

$$\text{Input power} = \frac{\text{shaft power}}{\text{efficiency of motor and drive}} \qquad (12.5)$$

Example 12.2 If the compressor of Example 12.1 is to be driven at 300 rev/min and is a single-acting, single-cylinder machine, calculate the cylinder bore required, assuming a stroke to bore ratio of 1.5/1. Calculate the power of the motor required to drive the compressor if the mechanical efficiency of the compressor is 85% and that of the motor transmission is 90%.

Solution Volume dealt with per unit time at inlet = 1 m^3/min

therefore

$$\text{Volume drawn in per cycle} = \frac{1}{300} = 0.003\,33 \text{ m}^3\text{/cycle}$$

i.e. $$\text{Cylinder volume} = 0.003\,33 \text{ m}^3$$

therefore

$$\frac{\pi}{4}d^2 L = 0.003\,33$$

where d is the bore and L the stroke,

i.e. $$\frac{\pi}{4}d^2(1.5 \times d) = 0.003\,33$$

therefore

$$d^3 = 0.002\,83\ \text{m}^3$$

i.e. Cylinder bore = 141.4 mm

$$\text{Power input to the compressor} = \frac{4.238}{0.85} = 4.99\ \text{kW}$$

therefore

$$\text{Motor power} = \frac{4.99}{0.9} = 5.54\ \text{kW}$$

Proceeding from equation (12.2), other expressions for the indicated work can be derived, i.e.

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}R(T_2 - T_1) = \frac{n}{n-1}\dot{m}RT_1\left(\frac{T_2}{T_1} - 1\right)$$

Also from equation (3.29)

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

Therefore

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}RT_1\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\} \tag{12.6}$$

or

$$\text{Indicated power} = \frac{n}{n-1}p_1\dot{V}\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\} \tag{12.7}$$

where $\dot{V}$ is the volume induced per unit time.

The condition for minimum work

The work done on the gas is given by the area of the indicator diagram, and the work done will be a minimum when the area of the diagram is a minimum. The height of the diagram is fixed by the required pressure ratio (when p_1 is fixed), and the length of the line da is fixed by the cylinder volume, which is itself fixed by the required induction of gas. The only process which can influence the area of the diagram is the line ab. The position taken by this line is decided by the value of the index n; Fig. 12.4 shows the limits of the possible processes.

Line ab_1 is according to the law pV = constant (i.e. isothermal)

Line ab_2 is according to the law pV^γ = constant (i.e. isentropic)

Both processes are reversible.

Isothermal compression is the most desirable process between a and b, giving the minimum work to be done on the gas. This means that in an actual

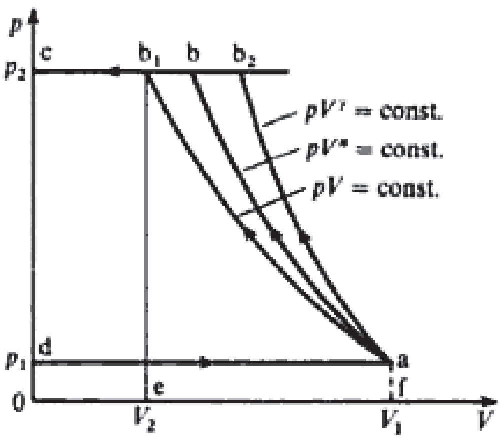


Fig. 12.4 Possible compression processes on a p–v diagram

compressor the gas temperature must be kept as close as possible to its initial value, and a means of cooling the gas is always provided, either by air or by water.

The indicated work done when the gas is compressed isothermally is given by the area ab_1cd.

$$\text{Area } ab_1cd = \text{area } ab_1ef + \text{area } b_1c0e - \text{area } ad0f$$

$$\text{Area } ab_1ef = p_2V_{b_1}\ln\frac{p_2}{p_1} \text{ (from equation (3.9))}$$

i.e. $$\text{indicated work per cycle} = p_2V_{b_1}\ln\frac{p_2}{p_1} + p_1V_{b_1} - p_1V_a$$

Also $p_1V_a = p_2V_{b_1}$, since the process ab_1 is isothermal, therefore

$$\text{indicated work per cycle} = p_2V_{b_1}\ln\frac{p_2}{p_1}$$

$$= p_1V_a\ln\frac{p_2}{p_1} \qquad (12.8)$$

$$= mRT\ln\frac{p_2}{p_1} \qquad (12.9)$$

When m and V_a in equations (12.8) and (12.9) are the mass and volume induced per unit time, then these equations give the isothermal power.

Isothermal efficiency

By definition, based on the indicator diagram

$$\text{Isothermal efficiency} = \frac{\text{isothermal work}}{\text{indicated work}} \qquad (12.10)$$

Example 12.3 Using the data of Example 12.1 calculate the isothermal efficiency of the compressor.

Solution From equation (12.9)

$$\text{Isothermal power} = \dot{m}RT \ln\frac{p_2}{p_1} = \frac{1.226 \times 0.287 \times 288}{60} \times \ln\frac{7}{1.013}$$

$$= 3.265 \text{ kW}$$

From Example 12.1,

$$\text{Indicated power} = 4.238 \text{ kW}$$

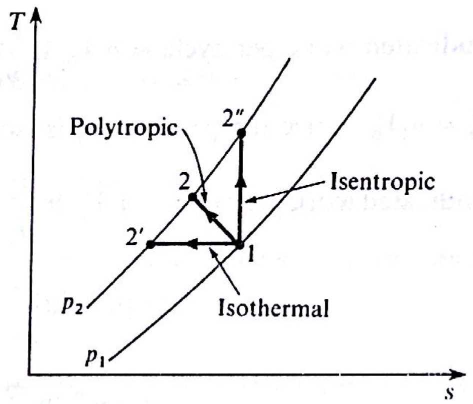
Therefore using equation (12.10) above

$$\text{Isothermal efficiency} = \frac{3.265}{4.238} = 0.77 \text{ or } 77\%$$

The least desirable form of compression in reciprocating compressors is that given by the isentropic process (see Fig. 12.4). The actual form of compression will usually be one between these two limits. The three processes are shown represented on a T–s diagram in Fig. 12.5:

1–2′ represents isothermal compression
1–2″ represents isentropic compression
1–2 represents compression according to a law pv^n = constant

Fig. 12.5 Isothermal, polytropic, and isentropic compression processes on a T–s diagram



The value of n is usually between 1.2 and 1.3 for a reciprocating air compressor. The main method used for cooling the air is by surrounding the cylinder by a water jacket and designing for the best ratio of surface area to volume of the cylinder.

12.2 Reciprocating compressors including clearance

Clearance is necessary in a compressor to give mechanical freedom to the working parts and allow the necessary space for valve operations.

Figure 12.6 shows the ideal indicator diagram with the clearance volume included. For good-quality machines the clearance volume is about 6% of the swept volume, and with a sleeve-valve machine it can be as low as 2%, but machines with clearances of 30–35% are also common.

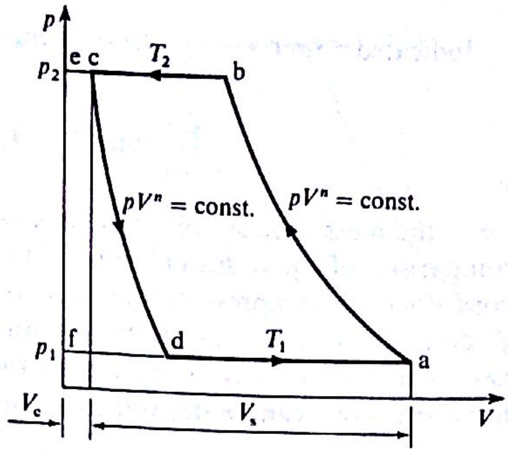


Fig. 12.6 Ideal indicator diagram for a reciprocating compressor with clearance

When the delivery stroke bc is completed the clearance volume V_c is full of gas at pressure p_2 and temperature T_2. As the piston proceeds on the next induction stroke the air expands behind it until the pressure p_1 is reached. Ideally as soon as the pressure reaches p_1, the induction of fresh gas will begin and continue to the end of this stroke at a. The gas is then compressed according to the law $pV^n = C$, and delivery begins at b as controlled by the valves. The effect of clearance is to reduce the induced volume at p_1 and T_1 from V_s to $(V_a - V_d)$. The masses of gas at the four principal points are such that $\dot{m}_a = \dot{m}_b$ and $\dot{m}_c = \dot{m}_d$. The mass delivered per unit time is given by $(\dot{m}_b - \dot{m}_c)$, which is equal to that induced, given by $(\dot{m}_a - \dot{m}_d)$. The properties of the working fluid change in processes a–b and c–d as shown in Fig. 12.7.

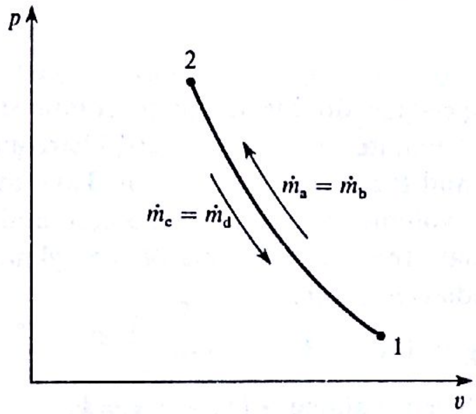


Fig. 12.7 Compression and re-expansion of masses of gas in a reciprocating compressor

Referring to Fig. 12.6 the indicated work done is given by the area of the p–V diagram.

$$\text{Indicated work} = \text{area abcd}$$

$$= \text{area abef} - \text{area cefd}$$

Then, using equation (12.2)

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}_a R(T_2 - T_1) - \frac{n}{n-1}\dot{m}_d R(T_2 - T_1)$$

i.e. $$\text{Indicated power} = \frac{n}{n-1} R(\dot{m}_a - \dot{m}_d)(T_2 - T_1)$$

$$= \frac{n}{n-1} R\dot{m}(T_2 - T_1) \tag{12.11}$$

where $\dot{m}$ is the mass induced per unit time $= (\dot{m}_a - \dot{m}_d)$.

A comparison of equations (12.11) and (12.2) shows that they are identical. The work done on compressing the mass of gas $\dot{m}_c$ (or $\dot{m}_d$) on compression, a–b, is returned when the gas expands from c to d. Hence the work done per unit mass of air delivered is unaffected by the size of the clearance volume.

Other expressions can be derived as before. From equation (12.7)

$$\text{Indicated power} = \frac{n}{n-1} p_1 \dot{V}\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\}$$

Also, if there are f cycles per unit time, then we have:

$$\dot{V} = f(V_a - V_d) \tag{12.12}$$

therefore

$$\text{Indicated power} = \frac{n}{n-1} p_1 f(V_a - V_d)\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\} \tag{12.13}$$

The mass delivered per unit time can be increased by designing the machine to be double acting, i.e. gas is dealt with on both sides of the piston, the induction stroke for one side being the compression stroke for the other (see Fig. 12.1).

Example 12.4 A single-stage, double-acting air compressor is required to deliver 14 m^3 of air per minute measured at 1.013 bar and 15 °C. The delivery pressure is 7 bar and the speed 300 rev/min. Take the clearance volume as 5% of the swept volume with a compression and re-expansion index of $n = 1.3$. Calculate the swept volume of the cylinder, the delivery temperature, and the indicated power.

Solution Referring to Fig. 12.8

$$\text{Swept volume} = (V_a - V_c) = V_s$$

and $$\text{Clearance volume, } V_c = 0.05V_s$$

i.e. $$V_a = 1.05V_s$$

Using equation (12.12) for a double-acting machine

$$\text{Volume induced per cycle, } (V_a - V_d) = \frac{14}{300 \times 2}$$

$$= 0.0233 \text{ m}^3/\text{cycle}$$

(cycles per minute = revolutions per minute × cycles per revolution).

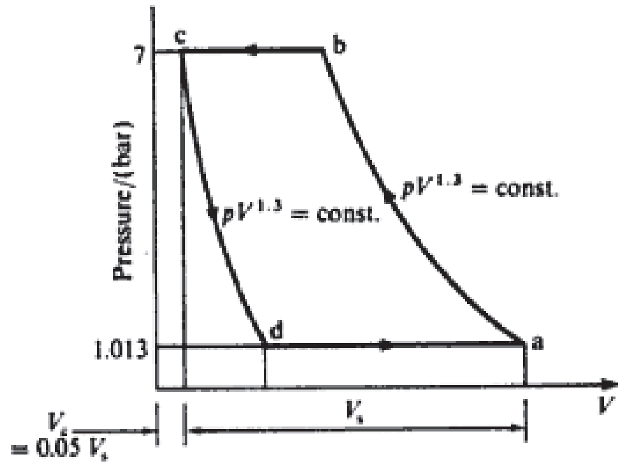


Fig. 12.8 Pressure–volume diagram for Example 12.4

Now

$$V_d = V_c\left(\frac{p_2}{p_1}\right)^{1/n} = 0.05V_s\left(\frac{7}{1.013}\right)^{1/1.3}$$

i.e. $V_d = 0.221V_s$

therefore

$$(V_a - V_d) = 1.05V_s - 0.221V_s = 0.0233 \text{ m}^3\text{/cycle}$$

therefore

$$V_s = \frac{0.0233}{0.829} = 0.0281 \text{ m}^3\text{/cycle}$$

i.e. Swept volume of compressor = 0.0281 m^3

Delivery temp., $T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n}$ from equation (3.29)

and $T_1 = 15 + 273 = 288$ K

i.e.

$$T_2 = 288\left(\frac{7}{1.013}\right)^{(1.3-1)/1.3}$$

$$= 450 \text{ K}$$

therefore

Delivery temp. = 177 °C

Using equation (12.7)

Indicated power

$$= \frac{n}{n-1}p_1\dot{V}\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right\}$$

$$= \frac{1.3}{0.3} \times \frac{1.013 \times 10^5 \times 14}{10^3 \times 60}\left\{\left(\frac{7}{1.013}\right)^{(1.3-1)/1.3} - 1\right\} \text{kW}$$

i.e. Indicated power = 57.6 kW

The approach used for a particular problem depends on how the data are stated and the quantities evaluated during the solution. In some problems it is better to evaluate $\dot{m}$ and T_2 and then use equation (12.11) for the indicated power; e.g. in Example 12.4 above, T_2 has been calculated, and the mass induced is given by

$$\dot{m} = \frac{1.013 \times 14 \times 10^5}{0.287 \times 288 \times 10^3} = 17.16 \text{ kg/min}$$

Then, using equation (12.11)

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}R(T_2 - T_1)$$

$$= \frac{1.3 \times 17.16 \times 0.287(450 - 288)}{0.3 \times 60}$$

$$= 57.6 \text{ kW} \quad \text{(as before)}$$

The diagrams previously shown (e.g. Fig. 12.8) are ideal diagrams. An actual indicator diagram is similar to the ideal except for the induction and delivery processes which are modified by a valve action. This is shown in Fig. 12.9. The waviness of the lines d–a and b–c is due to valve bounce. Automatic valves are in general use (see Fig. 12.1), and these are less definite in action than cam-operated valves; they also give more throttling of the gas. The induction stroke d–a is a mixing process, the induced air mixing with that in the cylinder.

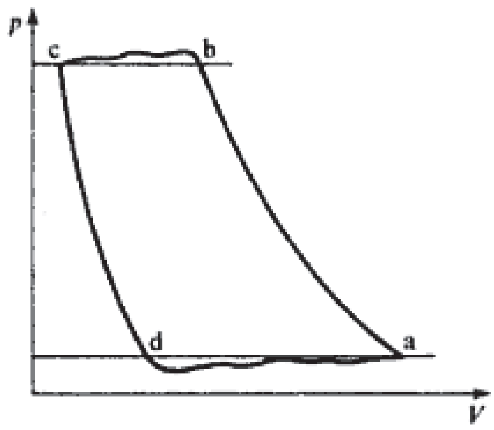


Fig. 12.9 Actual indicator diagram for a reciprocating compressor

Volumetric efficiency, η_v

It has been shown that one of the effects of clearance is to reduce the induced volume to a value less than that of the swept volume. This means that for a required induction the cylinder size must be increased over that calculated on the assumption of zero clearance. The volumetric efficiency is defined as follows:

η_v = the mass of gas delivered, divided by the mass of gas which would fill the swept volume at the free air conditions of pressure and temperature (12.14)

or

η_v = the volume of gas delivered measured at the free air pressure and temperature, divided by the swept volume of the cylinder (12.15)

The volume of air dealt with per unit time by an air compressor is quoted as the free air delivery (FAD), and is the rate of volume flow delivered, measured at the pressure and temperature of the atmosphere in which the machine is situated.

Equations (12.14) and (12.15) can be shown to be identical, i.e. if the FAD per cycle is V, at p and T, then the mass delivered per cycle is

$$m = \frac{pV}{RT}$$

The mass required to fill the swept volume, V_s, at p and T is given by

$$m_s = \frac{pV_s}{RT}$$

Therefore by equation (12.14),

$$\eta_v = \frac{m}{m_s} = \frac{pV}{RT} \times \frac{RT}{pV_s} = \frac{V}{V_s}.$$

The volumetric efficiency can be obtained from the indicator diagram. Referring to Fig. 12.10

$$\text{Volume induced} = V_a - V_d = V_s + V_c - V_d$$

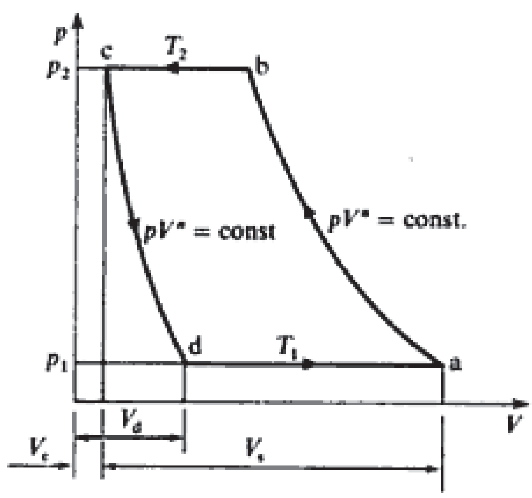


Fig. 12.10 Indicator diagram for a reciprocating compressor

and using equation (3.25)

$$\frac{V_d}{V_c} = \left(\frac{p_2}{p_1}\right)^{1/n} \quad \text{i.e. } V_d = V_c\left(\frac{p_2}{p_1}\right)^{1/n}$$

therefore

$$\text{Volume induced} = V_s + V_c - V_c\left(\frac{p_2}{p_1}\right)^{1/n}$$

$$= V_s - V_c\left\{\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right\}$$

Hence using equation (12.15),

$$\eta_v = \frac{V_a - V_d}{V_s} = \frac{V_s - V_c\{(p_2/p_1)^{1/n} - 1\}}{V_s}$$

i.e.

$$\eta_v = 1 - \frac{V_c}{V_s}\left\{\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right\} \tag{12.16}$$

It is important to note that this definition of volumetric efficiency is only consistent with that of equations (12.14) and (12.15) if the conditions of pressure and temperature in the cylinder during the induction stroke are identical with those of the free air. In fact the gas will be heated by the cylinder walls, and there will be a reduction in pressure due to the pressure drop required to induce the gas into the cylinder against the resistance to flow. These modifications to the ideal case require a more careful application of the formulae previously derived.

For example, when the FAD per cycle is denoted by V at p and T, then

$$m = \frac{pV}{RT} = \frac{p_1(V_a - V_d)}{RT_1}$$

i.e.

$$\text{FAD/cycle, } V = (V_a - V_d)\frac{T}{T_1}\frac{p_1}{p} \tag{12.17}$$

where p_1 and T_1 are the suction conditions.

Example 12.5 A single-stage, double-acting air compressor has a FAD of 14 m^3/min measured at 1.013 bar and 15 °C. The pressure and temperature in the cylinder during induction are 0.95 bar and 32 °C. The delivery pressure is 7 bar and the index of compression and expansion, n, is equal to 1.3. Calculate the indicated power required and the volumetric efficiency. The clearance volume is 5% of the swept volume.

Solution The p–V diagram is shown in Fig. 12.11

$$\text{Mass delivered per unit time, } \dot{m} = \frac{p\dot{V}}{RT}$$

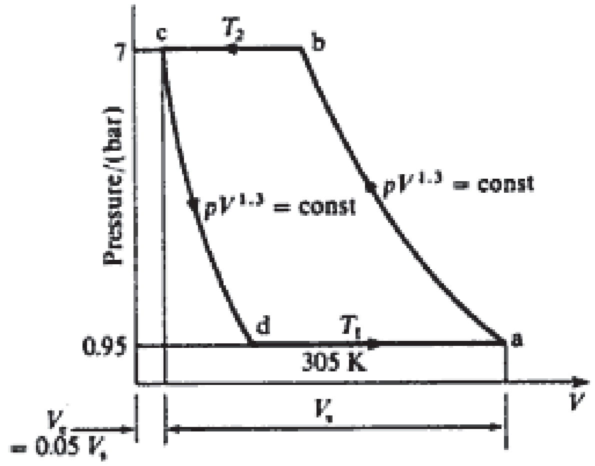


Fig. 12.11 Pressure–volume diagram for Example 12.5

where the FAD is $\dot{V}$ at p and T,

i.e. $$\dot{m} = \frac{1.013 \times 14 \times 10^5}{0.287 \times 288 \times 10^3} = 17.16 \text{ kg/min}$$

where $T = 15 + 273 = 288$ K.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n} \quad \text{from equation (3.29)}$$

i.e. $$T_2 = 305 \times \left(\frac{7}{0.95}\right)^{(1.3-1)/1.3} = 483.6 \text{ K}$$

where $T_1 = 32 + 273 = 305$ K.

From equation (12.11)

$$\text{Indicated power} = \frac{n}{n-1}\dot{m}R(T_2 - T_1)$$

$$= \frac{1.3 \times 17.16 \times 0.287(483.6 - 305)}{0.3 \times 60}$$

$$= 63.5 \text{ kW}$$

As before

$$V_d = V_c\left(\frac{p_2}{p_1}\right)^{1/n}$$

i.e. $$V_d = 0.05V_s\left(\frac{7}{0.95}\right)^{1/1.3} = 0.05V_s \times 7.368^{0.769}$$

$$= 0.232V_s$$

therefore

$$V_a - V_d = V_a - 0.232V_s = 1.05V_s - 0.232V_s = 0.818V_s$$

Using equation (12.17)

$$\text{FAD/cycle} = (V_a - V_d)\frac{T p_1}{T_1 p}$$

i.e. $$\text{FAD/cycle} = 0.818V_s \times \frac{288}{305} \times \frac{0.95}{1.013} = 0.724V_s$$

Then from equation (12.15)

$$\eta_v = \frac{V}{V_s} = \frac{0.724V_s}{V_s} = 0.724 \text{ or } 72.4\%$$

Note that if the volumetric efficiency in the above example is evaluated using equation 12.16 then

$$\eta_v = 1 - \frac{V_c}{V_s}\left\{\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right\} = 1 - \frac{0.05V_s}{V_s}\left\{\left(\frac{7}{0.95}\right)^{1/1.3} - 1\right\}$$

$$= 0.818 \text{ or } 81.8\%$$

There is a considerable difference between the two values, since the latter answer ignores the difference in temperature and pressure between the free air conditions and the suction conditions.

12.3 Multistage compression

It is shown in section 12.1 that the condition for minimum work is that the compression process should be isothermal. In general the temperature after compression is given by equation (3.29), $T_2 = T_1(p_2/p_1)^{(n-1)/n}$. The delivery temperature increases with the pressure ratio. Further, from equation (12.16)

$$\eta_v = 1 - \frac{V_c}{V_s}\left\{\left(\frac{p_2}{p_1}\right)^{1/n} - 1\right\}$$

it can be seen that as the pressure ratio increases the volumetric efficiency decreases. This is illustrated in Fig. 12.12.

For compression from p_1 to p_2 the cycle is abcd and the FAD per cycle is $V_a - V_d$; for compression from p_1 to p_3 the cycle is ab'c'd' and the FAD per cycle is $V_a - V_{d'}$; for compression from p_1 to p_4 the cycle is ab"c"d" and the FAD per cycle is $V_a - V_{d''}$. Therefore for a required FAD the cylinder size would have to increase as the pressure ratio increases.

The volumetric efficiency can be improved by carrying out the compression in two stages. After the first stage of compression the fluid is passed into a smaller cylinder in which the gas is compressed to the required final pressure. If the machine has two stages, the gas will be delivered at the end of this stage, but it could be delivered to a third cylinder for higher pressure ratios. The cylinders of the successive stages are proportioned to take the volume of gas delivered from the previous stage.

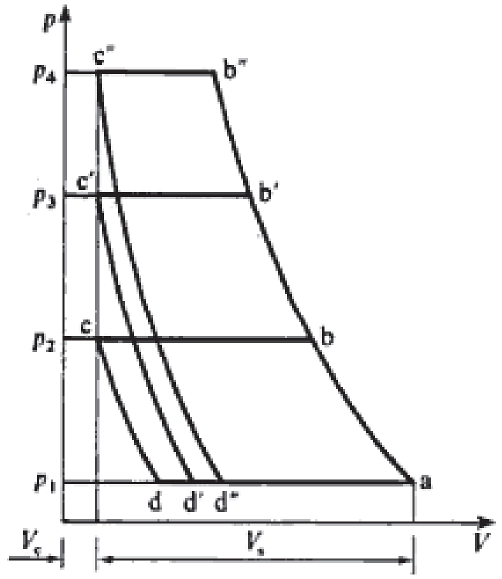


Fig. 12.12 Effect on the volumetric efficiency of increasing the delivery pressure

The indicator diagram for a two-stage machine is shown in Fig. 12.13. In this diagram it is assumed that the delivery process from the first or LP stage and the induction process of the second or HP stage are at the same pressure.

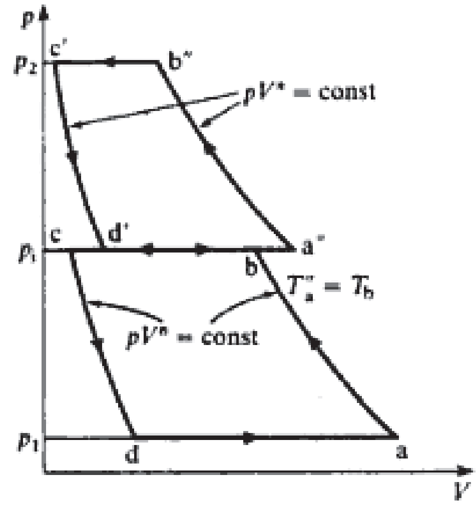


Fig. 12.13 Pressure–volume diagram for two-stage compression

The ideal isothermal compression can only be obtained if ideal cooling is continuous. This is difficult to obtain during normal compression. With multistage compression the opportunity presents itself for the gas to be cooled as it is being transferred from one cylinder to the next, by passing it through an intercooler. If intercooling is complete, the gas will enter the second stage at the same temperature at which it entered the first stage. The saving in work obtained by intercooling is shown by the shaded area in Fig. 12.14 and the diagram of the plant is shown in Fig. 12.15. The two indicator diagrams abcd and a'b'c'd' are shown with a common pressure, p_i. This does not occur in a real machine as there is a small pressure drop between the cylinders. An after-cooler can be fitted after the delivery process to cool the gas.

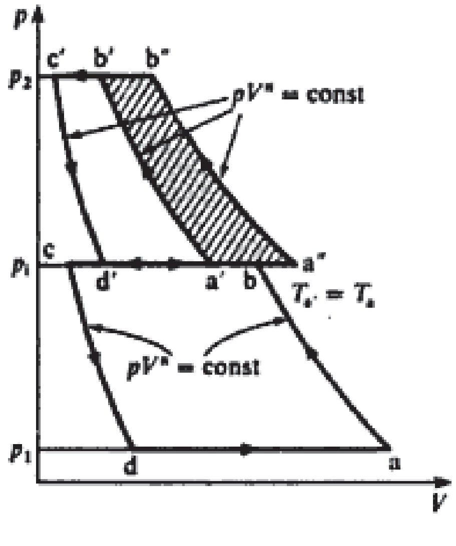


Fig. 12.14 Effect of intercooling on the compression work

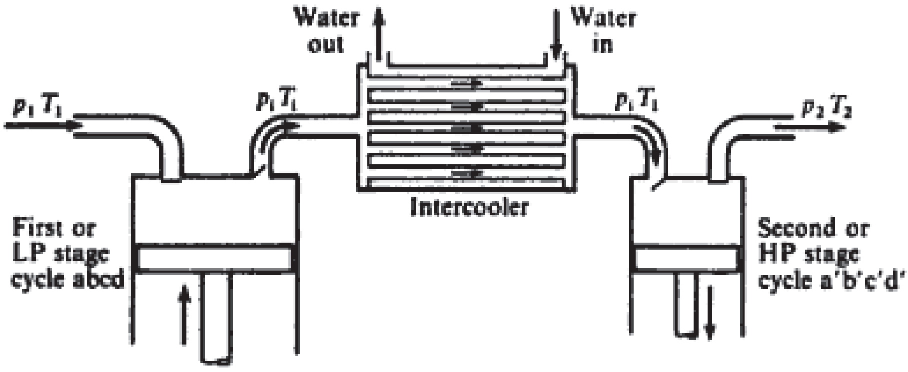


Fig. 12.15 Plan showing intercooling between compressor stages

The delivery temperatures from the two stages are given by

$$T_i = T_1\left(\frac{p_i}{p_1}\right)^{(n-1)/n} \quad \text{and} \quad T_2 = T_1\left(\frac{p_2}{p_i}\right)^{(n-1)/n}$$

respectively. This assumes that the gas is cooled in the intercooler back to the inlet temperature, and is called complete intercooling. To calculate the indicated power the equations (12.11) or (12.13) can be applied to each stage separately and the results added together. Two-stage compression with complete intercooling and after-cooling, and equal pressure ratios in each stage, is represented on a T–s diagram in Fig. 12.16.

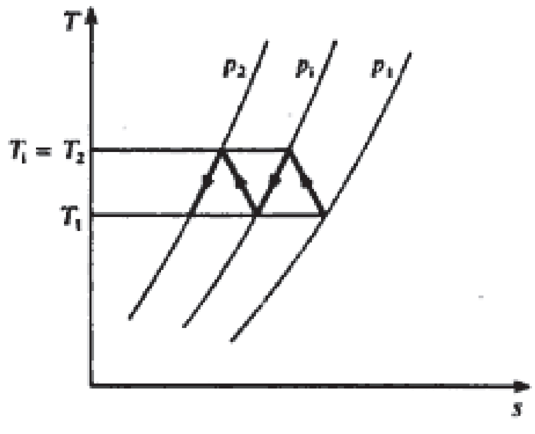


Fig. 12.16 T–s diagram showing intercooling and aftercooling

Example 12.6 In a single-acting, two-stage reciprocating air compressor 4.5 kg of air per minute are compressed from 1.013 bar and 15 °C through a pressure ratio of 9 to 1. Both stages have the same pressure ratio, and the law of compression and expansion in both stages is $pV^{1.3}$ = constant. If intercooling is complete, calculate the indicated power and the cylinder swept volumes required. Assume that the clearance volumes of both stages are 5% of their respective swept volumes and that the compressor runs at 300 rev/min.

Solution The two indicator diagrams are shown superimposed in Fig. 12.17. The LP stage cycle is abcd and the HP cycle is a'b'c'd'.

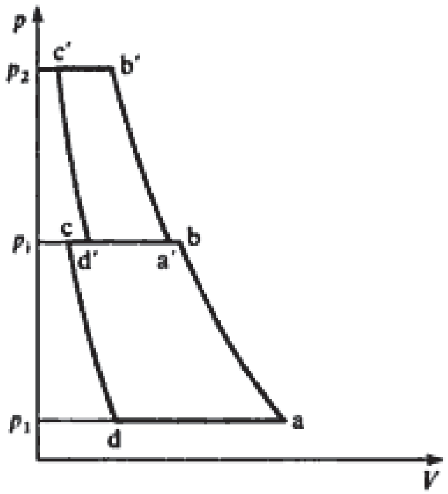


Fig. 12.17 Pressure–volume diagram showing both stages for Example 12.7

Now $p_2 = 9p_1$, also $p_i/p_1 = p_2/p_i$, therefore

$$p_i^2 = p_1 p_2 = 9p_1^2$$

therefore

$$p_i/p_1 = \sqrt{9} = 3$$

Using equation (3.29)

$$\frac{T_i}{T_1} = \left(\frac{p_i}{p_1}\right)^{(n-1)/n} \quad \text{therefore } \frac{T_i}{288} = 3^{(1.3-1)/1.3}$$

where $T_1 = 15 + 273 = 288$ K, and T_i is the temperature of the air entering the intercooler,

i.e. $\quad T_i = 288 \times 1.289 = 371$ K

Now as n, $\dot{m}$, and the temperature difference are the same for both stages, then the work done in each stage is the same. Therefore using equation (12.11)

$$\text{Total indicated power} = 2 \times \frac{n}{n-1}\dot{m}R(T_i - T_1)$$

$$= \frac{2 \times 1.3 \times 4.5 \times 0.287(371 - 288)}{0.3 \times 60}$$

$$= 15.5 \text{ kW}$$

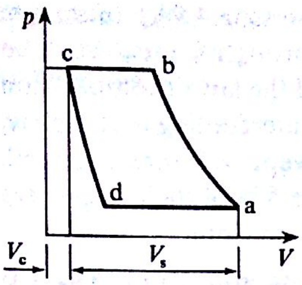


Fig. 12.18 Pressure–volume diagram for LP stage for Example 12.6

The mass induced per cycle is

$$\frac{4.5}{300} = 0.015 \text{ kg/cycle}$$

This mass is passed through each stage in turn.

For the LP cylinder, referring to Fig. 12.18,

$$V_a - V_d = \frac{mRT_1}{p_1} = \frac{0.015 \times 287 \times 288}{1.013 \times 10^5} = 0.0122 \text{ m}^3\text{/cycle}$$

Using equation (12.16)

$$\eta_v = \frac{V_a - V_c}{V_s} = 1 - \frac{V_c}{V_s}\left\{\left(\frac{p_i}{p_1}\right)^{1/n} - 1\right\} = 1 - 0.05(3^{1/1.3} - 1)$$

therefore

$$\eta_v = 1 - 0.066 = 0.934$$

Then

$$V_s = \frac{V_a - V_d}{0.934} = \frac{0.0122}{0.934} = 0.0131 \text{ m}^3\text{/cycle}$$

i.e. Swept volume of LP cylinder $= 0.0131 \text{ m}^3$

For the HP stage, a mass of 0.015 kg/cycle is drawn in at 15 °C and a pressure of $p_i = 3 \times 1.013 = 3.039$ bar, therefore

$$\text{Volume drawn in} = \frac{0.015 \times 287 \times 288}{3.039 \times 10^5}$$

$$= 0.004\,06 \text{ m}^3\text{/cycle}$$

Using equation (12.16) for the HP stage

$$\eta_v = 1 - \frac{V_c}{V_s}\left\{\left(\frac{p_2}{p_i}\right)^{1/n} - 1\right\}$$

and since V_c/V_s is the same as for the LP stage and also $p_2/p_i = p_i/p_1$ then η_v is 0.934 as above. Therefore

$$\text{Swept volume of HP stage} = \frac{0.004\,06}{0.934} = 0.004\,36 \text{ m}^3$$

Note that the clearance ratio is the same in each cylinder, and the suction temperatures are the same since intercooling is complete, therefore the swept volumes are in the ratio of the suction pressures,

i.e. $$V_{s_H} = \frac{V_{s_L}}{3} = \frac{0.0131}{3} = 0.004\,36 \text{ m}^3 \quad \text{(as above)}$$

The ideal intermediate pressure

The value chosen for the intermediate pressure p_i influences the work to be done on the gas and its distribution between the stages. The condition for the work done to be a minimum will be proved for two-stage compression but can be extended to any number of stages.

Total work = LP stage work + HP stage work.

Therefore using equation (12.6)

$$\text{Total power} = \frac{n}{n-1}\dot{m}RT_1\left\{\left(\frac{p_i}{p_1}\right)^{(n-1)/n} - 1\right\} + \frac{n}{n-1}\dot{m}RT_1\left\{\left(\frac{p_2}{p_i}\right)^{(n-1)/n} - 1\right\} \tag{12.18}$$

It is assumed that intercooling is complete and therefore the temperature at the start of each stage is T_1.

i.e. $$\text{Total power} = \frac{n}{n-1}\dot{m}RT_1\left\{\left(\frac{p_i}{p_1}\right)^{(n-1)/n} - 1 + \left(\frac{p_2}{p_i}\right)^{(n-1)/n} - 1\right\} \tag{12.19}$$

If p_1, T_1, and p_2 are fixed, then the optimum value of p_i which makes the power a minimum can be obtained by equating d (power)/(dp_i) to zero, i.e. optimum value of p_i when

$$\frac{d}{dp_i}\left\{\left(\frac{p_i}{p_1}\right)^{(n-1)/n} + \left(\frac{p_2}{p_i}\right)^{(n-1)/n} - 2\right\} = 0$$

i.e. when

$$\frac{d}{dp_i}\left\{\left(\frac{1}{p_1}\right)^{(n-1)/n} p_i^{(n-1)/n} + p_2^{(n-1)/n}\left(\frac{1}{p_i}\right)^{(n-1)/n} - 2\right\} = 0$$

therefore

$$p_1^{-(n-1)/n}\left(\frac{n-1}{n}\right)p_i^{\{(n-1)/n\}-1} + p_2^{(n-1)/n}\left(\frac{1-n}{n}\right)p_i^{\{(1-n)/n\}-1} = 0$$

therefore

$$p_1^{-(n-1)/n}\left(\frac{n-1}{n}\right)p_i^{-1/n} = p_2^{(n-1)/n}\left(\frac{n-1}{n}\right)p_i^{(1-2n)/n}$$

therefore

$$p_i^{\{2(n-1)\}/n} = (p_1 p_2)^{(n-1)/n}$$

therefore

$$p_i^2 = p_1 p_2 \tag{12.20}$$

or $$\frac{p_i}{p_1} = \frac{p_2}{p_i} \tag{12.21}$$

i.e. the pressure ratio is the same for each stage.

$$\text{Total minimum power} = 2 \times (\text{power required for one stage})$$

$$= 2 \times \frac{n\dot{m}RT_1}{n-1}\left\{\left(\frac{p_i}{p_1}\right)^{(n-1)/n} - 1\right\}$$

Or in terms of the overall pressure ratio p_2/p_1, we have, using equation (12.20),

$$\frac{p_i}{p_1} = \frac{\sqrt{p_1 p_2}}{p_1} = \sqrt{\frac{p_2}{p_1}}$$

therefore

$$\text{Total minimum power} = 2 \times \frac{n\dot{m}RT_1}{n-1}\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/2n} - 1\right\}$$

This can be shown to extend to z stages giving in general,

$$\text{Total minimum power} = z\frac{n}{n-1}\dot{m}RT_1\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/zn} - 1\right\} \qquad (12.22)$$

Also

$$\text{Pressure ratio for each stage} = \left(\frac{p_2}{p_1}\right)^{1/z} \qquad (12.23)$$

Hence the condition for minimum work is that the pressure ratio in each stage is the same and that intercooling is complete. (Note that in Example 12.6 the information given implies minimum work.)

Example 12.7 A three-stage, single-acting air compressor running in an atmosphere at 1.013 bar and 15 °C has a free air delivery of 2.83 m^3/min. The suction pressure and temperature are 0.98 bar and 32 °C respectively. Calculate the indicated power required, assuming complete intercooling, $n = 1.3$, and that the machine is designed for minimum work. The delivery pressure is to be 70 bar.

Solution

$$\text{Mass of air delivered} = \frac{pV}{RT} = \frac{1.013 \times 10^5 \times 2.83}{287 \times 288} = 3.47 \text{ kg/min}$$

where $T = 15 + 273 = 288$ K.

Then using equation (12.22)

Total indicated power

$$= z\frac{n}{n-1}\dot{m}RT_1\left\{\left(\frac{p_2}{p_1}\right)^{(n-1)/zn} - 1\right\}$$

$$= 3 \times \frac{1.3}{0.3} \times \frac{3.47 \times 0.287 \times 288}{60}\left\{\left(\frac{70}{0.98}\right)^{(1.3-1)/(3\times 1.3)} - 1\right\}$$

$$= 24.2 \text{ kW}$$

Besides the benefits of multistage compression already dealt with there are also mechanical advantages. The higher pressures are confined to the smaller cylinders and a multicylinder machine has less variation in rotational speed and requires a smaller flywheel.

Energy balance for a two-stage machine with intercooler

Referring to Fig. 12.19, the steady-flow energy equation (1.10) can be applied to the LP stage, the intercooler, and the HP stage, in turn. Changes in kinetic energy and height can be neglected, i.e. from equation (1.10)

$$h_1 + \frac{C_1^2}{2} + Q + W = h_2 + \frac{C_2^2}{2}$$

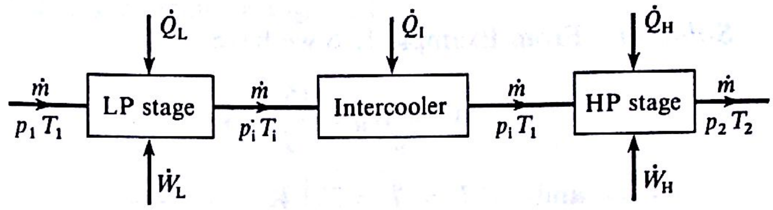


Fig. 12.19 Steady flow through a two-stage reciprocating compressor with intercooler

for the LP stage, for unit mass flow rate,

$$h_1 + Q_L + W_L = h_i$$

or for mass flow rate, $\dot{m}$

$$\dot{m}c_p T_1 + \dot{Q}_L + \dot{W}_L = \dot{m}c_p T_i$$

therefore

$$\dot{Q}_L = -\{\dot{W}_L - \dot{m}c_p(T_i - T_1)\}$$

i.e. Heat rejected in LP stage $= \dot{W}_L - \dot{m}c_p(T_i - T_1)$ (12.24)

for the intercooler, for unit mass flow rate

$$h_i + Q_I = h_1$$

or for mass flow rate, $\dot{m}$

$$\dot{m}c_p T_i + \dot{Q}_I = \dot{m}c_p T_1$$

therefore

$$\dot{Q}_I = -\dot{m}c_p(T_i - T_1)$$

i.e. Heat rejected in intercooler $= \dot{m}c_p(T_i - T_1)$ (12.25)

for the HP stage, for unit mass flow rate,

$$h_1 + Q_H + W_H = h_2$$

or for mass flow rate, $\dot{m}$

$$\dot{m}c_p T_1 + \dot{Q}_H + \dot{W}_H = \dot{m}c_p T_2$$

therefore

$$\dot{Q}_H = -\{\dot{W}_H - \dot{m}c_p(T_2 - T_1)\}$$

i.e. Heat rejected in HP stage $= \dot{W}_H - \dot{m}c_p(T_2 - T_1)$ (12.26)

With complete intercooling, as assumed in Fig. 12.19, and the compressor designed for minimum work, then, from equation (12.11),

$$\dot{W}_L = \dot{W}_H = \frac{n}{n-1}\dot{m}R(T_2 - T_1)$$

Example 12.8 Using the data of Example 12.6 determine the rate of heat loss to the cylinder jacket cooling water and the rate of heat loss to the intercooler circulating water.

Solution From Example 12.6 we have

$$\dot{W}_L = \dot{W}_H = \frac{15.5}{2}\text{ kW}$$

and $T_2 = T_i = 371$ K

Then, from equation (12.24)

$$-\dot{Q}_L = \dot{W}_L - \dot{m}c_p(T_2 - T_1)$$

therefore

$$-\dot{Q}_L = \frac{15.5}{2} - \frac{4.5 \times 1.005}{60}(371 - 288)$$

i.e. $-\dot{Q}_L = 7.75 - 6.26 = 1.49$ kW

From equation (12.26)

$$-\dot{Q}_H = \dot{W}_H - \dot{m}c_p(T_2 - T_1)$$

and $\dot{W}_H = \dot{W}_L$ and $T_2 = T_i$

therefore

$$\dot{Q}_H = \dot{Q}_L = -1.49\text{ kW}$$

i.e. Heat loss from the cylinder in each stage = 1.49 kW

From equation (12.25)

$$-\dot{Q}_I = \dot{m}c_p(T_i - T_1) = \frac{4.5 \times 1.005}{60} \times (371 - 288)$$

i.e. Heat to intercooler circulating water = 6.26 kW

The quantities $\dot{W}_L$ and $\dot{W}_H$, as defined by Fig. 12.19, are the rates of work done on the air. The actual power inputs exceed this by the amounts necessary to overcome frictional resistance to the moving parts of the machine. It can be assumed that about 50% of the friction power goes to increasing the energy transferred to the cooling water, in addition to the heat transferred to the cooling water from the air in the cylinder.

12.4 Steady-flow analysis

In section 12.2 an expression was obtained (equation (12.11)) for the indicated power required to take a mass flow rate of gas, $\dot{m}$, in state 1 and deliver it at a higher pressure in state 2. This was done by analysing the internal processes of the machine. Another approach is to consider the compression process as one of steady flow, as shown in Fig. 12.20, with the change of state from 1 to 2 being achieved by a non-flow process of polytropic compression, as indicated in the property diagram of Fig. 12.21.

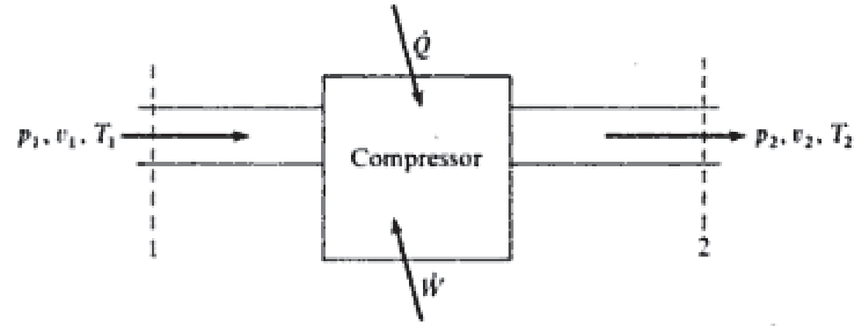


Fig. 12.20 Steady flow through a reciprocating compressor

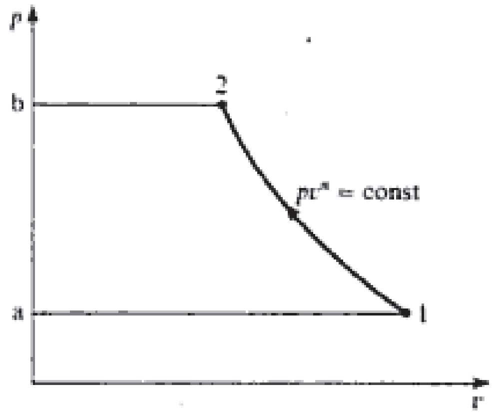


Fig. 12.21 Compression process on a p–v diagram

The steady-flow energy equation for the system shown in Fig. 12.20, neglecting changes in potential and kinetic energy and for unit mass flow rate, is

$$h_1 + Q + W = h_2$$

therefore

$$Q + W = h_2 - h_1$$

or for an elemental process

$$dQ + dW = dh \qquad \text{(a)}$$

Assuming that no heat is transferred on induction or delivery the heat transferred, to or from the system, takes place during the polytropic non-flow compression process. The non-flow equation for a reversible process states

$$dQ = du + p\,dv \tag{b}$$

Combining (a) and (b) gives

$$dh = du + p\,dv + dW$$

and, by definition, $h = u + pv$, hence $dh = du + p\,dv + v\,dp$, therefore substituting

$$du + p\,dv + v\,dp = du + p\,dv + dW$$

therefore

$$dW = v\,dp$$

Then

$$W = \int_1^2 v\,dp = \text{area 12ba1 in Fig. 12.21,}$$

i.e.

$$W = C^{1/n}\int_1^2 \frac{dp}{p^{1/n}} \quad \left(\text{since } v = \frac{C^{1/n}}{p^{1/n}} \text{ if } pv^n = C\right)$$

$$= C^{1/n}\left[\left(\frac{n}{n-1}\right)p^{(n-1)/n}\right]_1^2$$

$$= \left[\left(\frac{n}{n-1}\right)p^{(n-1)/n}p^{1/n}v\right]_1^2$$

$$= \left[\frac{n}{n-1}pv\right]_1^2$$

$$= \frac{n}{n-1}(p_2v_2 - p_1v_1)$$

i.e. Work input, $W = \dfrac{n}{n-1}(p_2v_2 - p_1v_1)$

and as $p_1v_1 = RT_1$ and $p_2v_2 = RT_2$ then

$$W = \frac{nR}{n-1}(T_2 - T_1)$$

12.5 Rotary machines

Because of the continuous rotary action, the rotary positive displacement machine is smaller for a given flow than its reciprocating counterpart. The machines in this category are generally uncooled and as the compression is carried out at a high rate the conditions are approximately adiabatic. Examples of this type are: (i) the Roots blower; (ii) vane type.

Roots blower

The two-lobe type is shown in Fig. 12.22, but three- and four-lobe versions are in use for higher pressure ratios. One of the rotors is connected to the drive and the second rotor is gear driven from the first. In this way the rotors rotate in phase and the profile of the lobes is of cycloidal or in volute form giving correct mating of the lobes to seal the delivery side from the inlet side. This sealing continues until delivery commences. There must be some clearance between the lobes and between the casing and the lobes to reduce wear; this clearance forms a leakage path which has an increasingly adverse effect on efficiency as the pressure ratio increases.

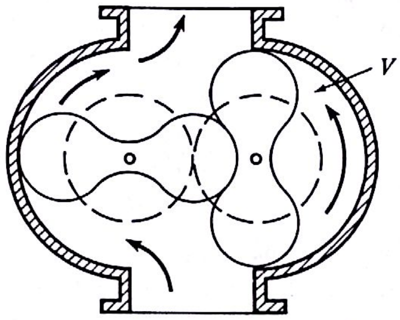


Fig. 12.22 Roots blower with a two-lobe rotor

As each side of each lobe faces its side of the casing a volume of gas V, at pressure p_1, is displaced towards the delivery side at constant pressure. A further rotation of the rotor opens this volume to the receiver, and the gas flows back from the receiver, since this gas is at a higher pressure. The gas induced is compressed irreversibly by that from the receiver to the pressure p_2, and then delivery begins. This process is carried out four times per revolution of the driving shaft.

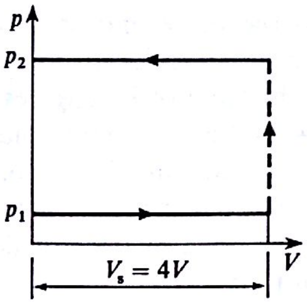


Fig. 12.23 Pressure–volume diagram for a Roots blower

The p–V diagram for this machine is shown in Fig. 12.23, in which the pressure rise from p_1 to p_2 is shown as an irreversible process at constant volume.

$$\text{Work done per cycle} = (p_2 - p_1)V$$

therefore

$$\text{Work done per revolution} = 4(p_2 - p_1)V \tag{12.27}$$

If $\dot{V}_s$ is the volume dealt with per unit time at p_1 and T_1, then

$$\text{Power input} = (p_2 - p_1)\dot{V}_s \tag{12.28}$$

The ideal compression process from p_1 to p_2 is a reversible adiabatic (i.e. isentropic) process. The work done per minute ideally is thus given by equation (12.7) with $n = \gamma$,

i.e. $$\text{Power input} = \frac{\gamma}{\gamma - 1} p_1 \dot{V}_s \left\{ \left(\frac{p_2}{p_1} \right)^{(\gamma-1)/\gamma} - 1 \right\}$$

Then a comparison may be made on the basis of a Roots efficiency,

i.e. $$\text{Roots efficiency} = \frac{\text{work done isentropically}}{\text{actual work done}}$$

i.e. $$\text{Roots efficiency} = \frac{\{\gamma/(\gamma-1)\}p_1\dot{V}_s\{(p_2/p_1)^{(\gamma-1)/\gamma}-1\}}{\dot{V}_s(p_2-p_1)}$$

$$= \frac{\gamma\{r^{(\gamma-1)/\gamma}-1\}}{(\gamma-1)(r-1)}$$

where r = pressure ratio, p_2/p_1.

From equation (2.22), we can write

$$\frac{\gamma}{\gamma-1} = \frac{c_p}{R}$$

therefore

$$\text{Roots efficiency} = \frac{c_p}{R}\left\{\frac{r^{(\gamma-1)/\gamma}-1}{(r-1)}\right\} \tag{12.29}$$

For a Roots air blower values of pressure ratio, r, of 1.2, 1.6, and 2 give values for the Roots efficiency of 0.945, 0.84, and 0.765 respectively. These values show that the efficiency decreases as the pressure ratio increases.

The actual compression process is not quite as simple as that described. When the displacement volume V is opened to the delivery space a pressure wave enters which increases with the opening and moves at the velocity of sound. This wave is reflected from the approaching lobe to the delivery space. The pressure oscillations set up unsteady conditions in the delivery space which vary considerably from one design to another. The actual torque and loading on the rotors are higher than is suggested by the p–V diagram, and fluctuate with high frequency. This fluctuation is transmitted to the drive and creates difficulties due to vibrations. This machine has a number of imperfections, but is well suited to such tasks as the scavenging and supercharging of IC engines.

Roots blowers are built for capacities of from 0.14 to 1400 m^3/min, and pressure ratios of the order of 2 to 1 for a single-stage machine and 3 to 1 for a two-stage machine. Other designs have been produced to improve on the Roots blower, one of these being the Bicera compressor, designed by the British Internal Combustion Engineering Research Association (BICERA).

Vane type

The simple vane type is shown in Fig. 12.24 and consists of a rotor mounted eccentrically in the body, and supported by ball- and roller-bearings in the end covers of the body. The rotor is slotted to take the blades which are of a non-metallic material, usually fibre or carbon. As each blade moves past the inlet passage, compression begins due to decreasing volume between the rotor and casing. Delivery begins with the arrival of each blade at the delivery passage.

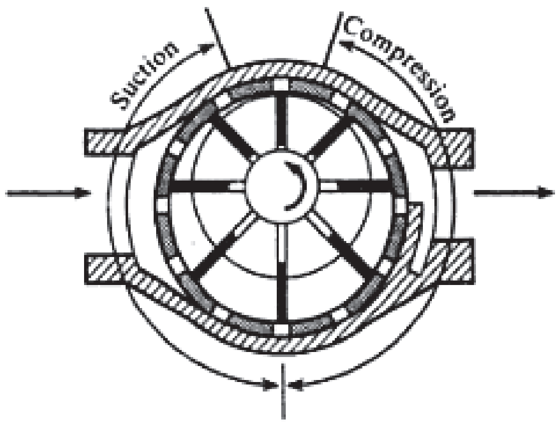


Fig. 12.24 Vane-type positive displacement compressor

This type of compression differs from that of the Roots blower in that some or all of the compression is obtained before the trapped volume is opened to delivery. Further compression can be obtained by the back-flow of air from the receiver which occurs in an irreversible manner.

The $p-V$ diagram is shown in Fig. 12.25. V_s is the induced volume at pressure p_1 and temperature T_1. Compression occurs to the pressure p_i, the ideal form for an uncooled machine being isentropic. At this pressure the displaced gas is opened to the receiver and gas flowing back from the receiver raises the pressure irreversibly to p_2. The work input is given by the sum of the areas A and B, referring to Fig. 12.25. Comparing the areas of Figs 12.23 and 12.25 it can be seen that for a given airflow and given pressure ratio the vane type requires less work input than the Roots blower.

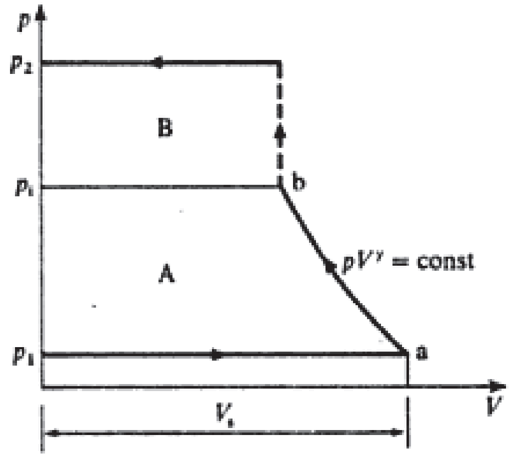


Fig. 12.25 Pressure–volume diagram for a vane-type compressor

A rotary sliding vane two-stage machine is shown in Fig. 12.26; in this type the vanes are in contact with the cylinder walls.

Example 12.9 Compare the work inputs required for a Roots blower and a vane-type compressor having the same induced volume of 0.03 m^3/rev, the inlet pressure being 1.013 bar and the pressure ratio 1.5 to 1. For the vane type assume that internal compression takes place through half the pressure range.

Solution $p_1 = 1.013$ bar

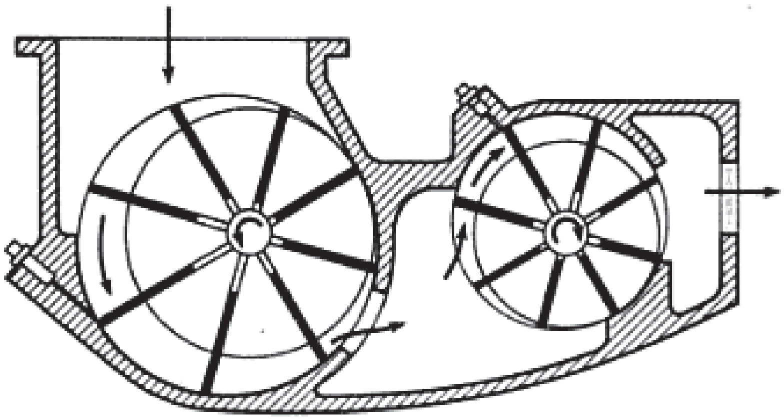


Fig. 12.26 Rotary sliding vane two-stage positive displacement compressor

therefore

$$p_2 = 1.013 \times 1.5 = 1.520 \text{ bar}$$

For the Roots blower, referring to Fig. 12.23

$$\text{Work done per revolution} = (p_2 - p_1)V_s$$

$$= (1.520 - 1.013) \times \frac{10^5 \times 0.03}{10^3}$$

$$= 1.52 \text{ kJ/rev}$$

For the vane type

$$p_i = \frac{(1.5 \times 1.013) + 1.013}{2} = 1.266 \text{ bar}$$

Referring to Fig. 12.25

$$\text{Work required} = (\text{area A} + \text{area B})$$

Now using equation (12.7) with $n = \gamma$

$$\text{area A} = \frac{\gamma}{\gamma - 1} p_1 V_s \left\{ \left(\frac{p_i}{p_1} \right)^{(\gamma - 1)/\gamma} - 1 \right\}$$

$$= \frac{1.4}{0.4} \times \frac{1.013 \times 10^5 \times 0.03}{10^3} \left\{ \left(\frac{1.266}{1.013} \right)^{0.4/1.4} - 1 \right\} \text{ kJ/rev}$$

$$= 0.70 \text{ kJ/rev}$$

$$\text{area B} = (p_2 - p_i)V_b$$

where V_b is given by equation (3.19),

i.e. $$V_b = V_s \left(\frac{p_1}{p_i} \right)^{1/\gamma} = 0.03 \times \left(\frac{1.013}{1.266} \right)^{1/1.4}$$

$$= 0.0256 \text{ m}^3$$

i.e. $$\text{area B} = (1.520 - 1.266) \times 10^2 \times 0.0256 \text{ kJ/rev}$$

$$= 0.65 \text{ kJ/rev}$$

therefore

$$\text{Work required} = 0.70 + 0.65 = 1.35\ \text{kJ/rev}$$

(compared with the work required for the Roots machine of 1.52 kJ/rev).

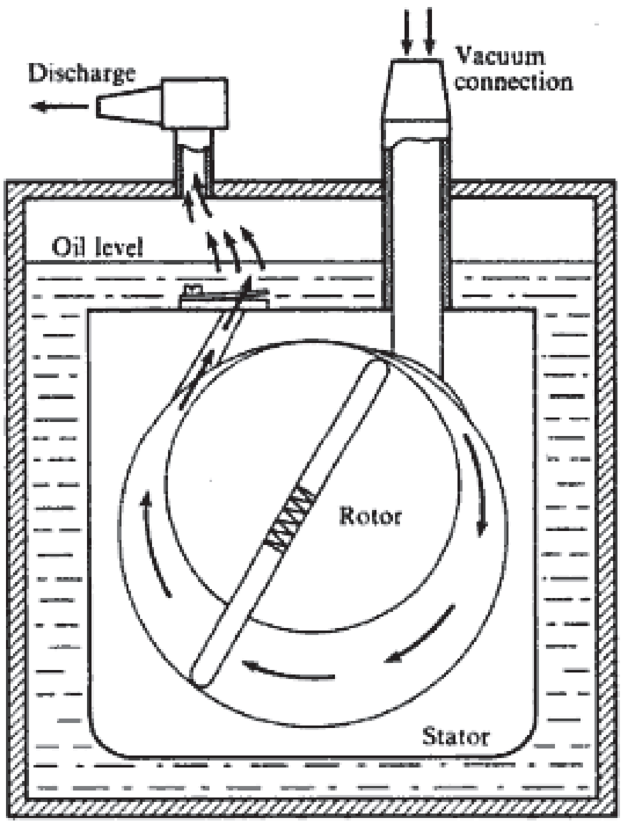
Rotary sliding vane compressors are used with free air deliveries of up to 150 m^3/min and pressure ratios up to 8.5 to 1. For special applications and boosting, pressure ratios of the order of 20 to 1 have been obtained from this type. The larger machines are usually water-cooled.

Lubrication is important with vane-type machines and is accomplished by injecting oil to the vane tips in contact with the casing. Some machines, having carbon vanes, require no lubrication. Another version is designed to reduce the friction between vane and casing. This employs a floating drum which rotates between the rotor and casing, and does not allow the vanes to make contact with the casing. The only movement of the blades relative to the floating drum is along the slots. See Fig. 12.26.

12.6 Vacuum pumps

Rotary positive displacement pumps are used to produce a vacuum or to scavenge a vessel. An example of this type of pump is shown in Fig. 12.27. The rotor is eccentrically mounted in the stator and carries two blades which sweep the space between the rotor and stator. The gas being exhausted enters through

Fig. 12.27 Rotary positive displacement vacuum pump



the vacuum connection and is compressed before delivery through the discharge valve. The efficiency of such pumps is impaired by the presence of condensable vapours, and means must be provided to deal with these if necessary. The vapours tend to condense before delivery through the discharge valve and mix with the sealing oil. The liquid eventually evaporates into the vacuum system and lowers the vacuum obtainable, as well as impairing the sealing and lubricating properties of the oil.

12.7 Air motors

Compressed air is used in a wide variety of applications in industry. For some purposes air-operated motors are the most suitable forms of power, especially where there are safety requirements to be met as in mining applications.

Pneumatic breakers, picks, spades, rammers, vibrators, riveters, etc. form a range of hand tools which have wide applications in constructional work. They are light in construction and suitable for operation in remote situations for which other forms of power tools may not be suitable. The action required of such tools, with the associated simplicity and robustness of construction, is obtained with air-operated design.

Basically the cycle in the reciprocating expander is the reverse of that in the reciprocating compressor. Air is supplied to the air motor from an air receiver in which the air is at approximately ambient temperature. There is a pressure drop in the air line between the receiver and the motor. The air expands in the motor cylinder to atmospheric pressure in a manner which is polytropic (i.e. the expansion is internally reversible and the law of expansion is pv^n = constant, where $n < \gamma$, and is usually about 1.3). If the air is initially at ambient temperature, then this form of expansion will bring about a reduction in the air temperature as lower pressures are reached. The temperatures reached may be sufficiently low to be below the dew-point of the moisture in the air (see section 15.2); this moisture may be condensed, and the water formed may even be cooled to its freezing-point. This may lead to the formation of ice in the cylinder with the consequence of blocked valves. To prevent this condition it may be necessary to preheat the air to an initial temperature which is high enough to prevent the formation of ice. This heating of the air causes an increase in volume at the supply pressure and reduces the demand from the compressor. Further, the temperature at which the heat transfer is required is low, and a low-grade supply of heat or 'waste heat' may be utilized for the purpose.

A hypothetical indicator diagram for an air motor is shown in Fig. 12.28. In this case the air expands from 1 to the pressure p_2 at the end of the stroke. There is then a *blow-down* of air from 2 to 3. Air is exhausted from 3 to 4, and at 4 compression of the trapped or *cushion air* begins. Air at the supply pressure, p_6, is admitted to the cylinder at the point 5 where it mixes irreversibly with the cushion air. The pressure in the cylinder is rapidly brought up to the inlet value, p_6. The further supply of air is made at constant pressure behind the

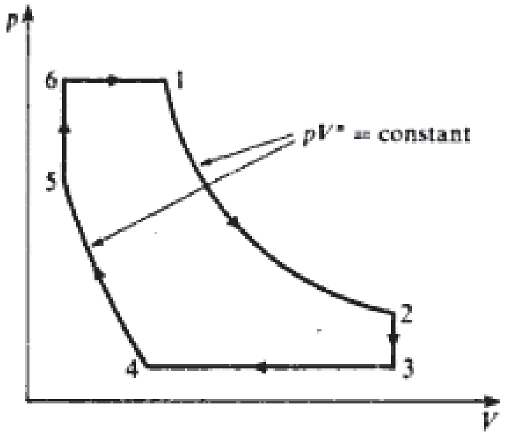


Fig. 12.28 Pressure–volume diagram for an air motor

moving piston to the point of cut-off at 1. The cut-off ratio is given by

$$\text{Cut-off ratio} = \frac{V_1 - V_6}{V_3 - V_6}$$

The effect of the cushion air is to give a smoother-running motor. The position of the point 5 depends on the point of initial compression 4, and on the law of compression pV^n = constant. The conditions may be such that the points 5 and 6 coincide.

The analysis of such a diagram is best carried out from basic principles, as illustrated in the following example.

Example 12.10 The cylinder of an air motor has a bore of 63.5 mm and a stroke of 114 mm. The supply pressure is 6.3 bar, the supply temperature 24 °C, and the exhaust pressure is 1.013 bar. The clearance volume is 5% of the swept volume and the cut-off ratio is 0.5. The air is compressed by the returning piston after it has travelled through 0.95 of its stroke. The law of compression and expansion is $pV^{1.3}$ = constant. Calculate:

(i) the temperature at the end of expansion;
(ii) the indicated power of the motor which runs at 300 rev/min;
(iii) the air supplied per minute.

Solution (i) Referring to the cycle of Fig. 12.28

$$\text{Clearance volume} = V_6 = V_5 = 0.05V_s$$

Since the cut-off ratio, $(V_1 - V_6)/(V_3 - V_6)$, is 0.5, therefore

$$V_1 = 0.5V_s + 0.05V_s = 0.55V_s$$

$$V_2 = V_s + V_6 = 1.05V_s$$

Now

$$V_3 - V_4 = 0.95(V_3 - V_5) \quad \text{(given)}$$

or $\quad V_4 - V_5 = 0.05V_s$

therefore

$$V_4 = 0.05V_s + 0.05V_s = 0.1V_s$$

$$p_1 V_1^n = p_2 V^n$$

therefore

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^n = 6.3\left(\frac{0.55}{1.05}\right)^{1.3} = 2.718 \text{ bar}$$

also $$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{n-1} = 297\left(\frac{0.55}{1.05}\right)^{0.3} = 244.6 \text{ K}$$

i.e. Temperature after expansion = 244.6 − 273 = −28.4 °C

(ii) Now

$$p_5 = p_4\left(\frac{V_4}{V_5}\right)^n = 1.013\left(\frac{0.1}{0.05}\right)^{1.3} = 2.494 \text{ bar}$$

Work output per cycle = area 1 234 561

$$\text{Work output} = p_1(V_1 - V_6) + \left(\frac{p_1V_1 - p_2V_2}{n-1}\right)$$

$$-p_3(V_3 - V_4) - \left(\frac{p_5V_5 - p_4V_4}{n-1}\right)$$

and $$\text{Swept volume} = \frac{\pi \times 63.5^2 \times 114}{4 \times 10^9} = 0.361 \times 10^{-3} \text{ m}^3$$

therefore

Work output per cycle

$$= (6.3 \times 10^5 \times 0.361 \times 10^{-3} \times 0.5)$$

$$+ \frac{10^5 \times 0.361 \times 10^{-3}}{0.3}\{(6.3 \times 0.55) - (2.718 \times 1.05)\}$$

$$- (1.013 \times 10^5 \times 0.361 \times 10^{-3} \times 0.95)$$

$$- \frac{10^5 \times 0.361 \times 10^{-3}}{0.3}\{(2.494 \times 0.05) - (1.013 \times 0.1)\}$$

i.e. Work output per cycle = 113.7 + 73.5 − 34.7 − 2.8

= 149.7 J/cycle

$$\text{Power developed} = \frac{149.7 \times 300}{60 \times 10^3} = 0.749 \text{ kW}$$

(iii) The mass induced per cycle is given by $(m_1 - m_4)$. It is necessary to determine the temperature of the air at 4, which can be taken as equal to that at 3. It is assumed that the air in the cylinder at the point 2 expands isentropically to the exhaust pressure. Therefore

$$T_3 = T_2\left(\frac{p_3}{p_2}\right)^{(\gamma - 1)/\gamma} = 244.6\left(\frac{1.013}{2.718}\right)^{0.4/1.4} = 184.5\ \text{K}$$

i.e.

$$m_4 = \frac{p_4 V_4}{RT_4} = \frac{1.013 \times 10^5 \times 0.0361 \times 10^{-3}}{287 \times 184.5} = 0.0691 \times 10^{-3}\ \text{kg}$$

$$m_1 = \frac{p_1 V_1}{RT_1} = \frac{6.3 \times 10^5 \times 0.55 \times 0.361 \times 10^{-3}}{287 \times 297} = 1.4675 \times 10^{-3}\ \text{kg}$$

therefore

$$\text{Induced mass per cycle} = (1.4675 \times 10^{-3}) - (0.0691 \times 10^{-3})$$
$$= 1.398 \times 10^{-3}\ \text{kg}$$

i.e. Mass flow rate of air supplied $= 1.398 \times 10^{-3} \times 300 = 0.42$ kg/min

---

Air motors can be rotary in action and are then similar in form to their compressor counterparts, see section 12.5. Figure 12.29(a), (b), and (c) show the forms of the performance characteristics of a small, 0.3 kW, vane type air motor in terms of power/speed, torque/speed, and air consumption/speed. An air motor which receives air from a constant pressure supply can be controlled to meet the load requirements by fitting a restrictor either before or after the motor. It can be shown by a consideration of a simplified p–V diagram, neglecting clearance, that fitting the restrictor before the motor requires a lower airflow than fitting it after. The reader should establish this for himself and also show that the airflow rate required is approximately proportional to the supply pressure to the motor for a given duty. Figure 12.30 shows the results of a test on a small air motor which gives a 25% reduction in air requirement if the restrictor is on the inlet side to the motor.

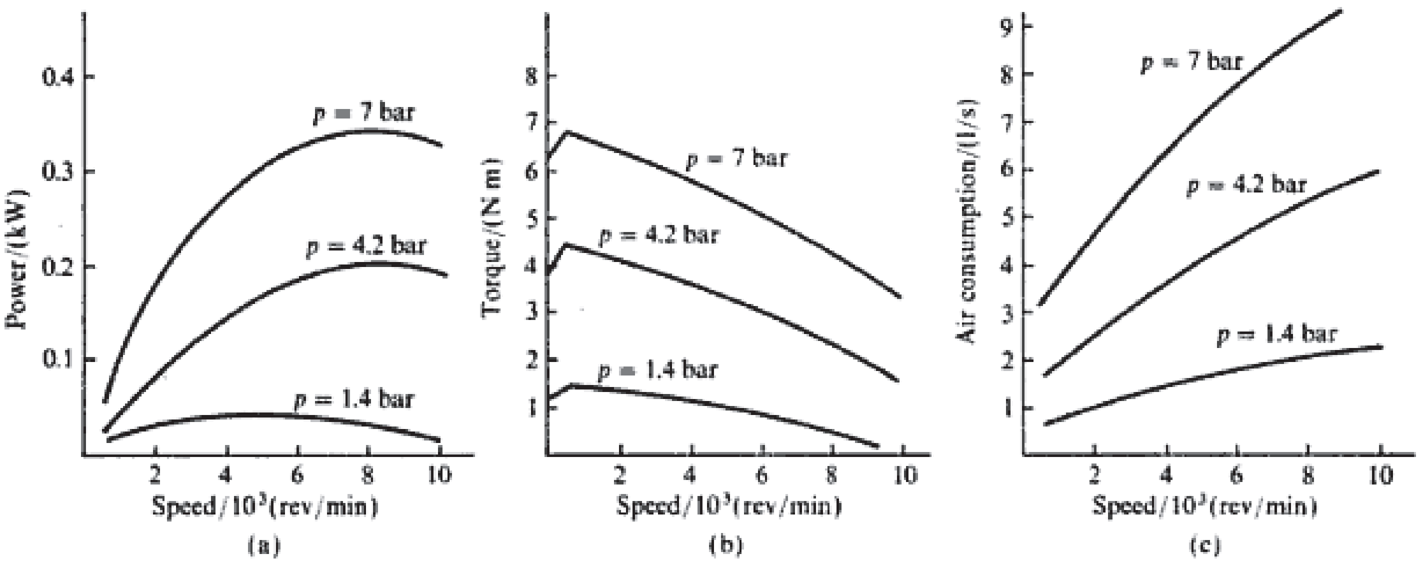


Fig. 12.29 Characteristics of a small vane-type air motor: (a) power–speed, (b) torque–speed, (c) air consumption–speed

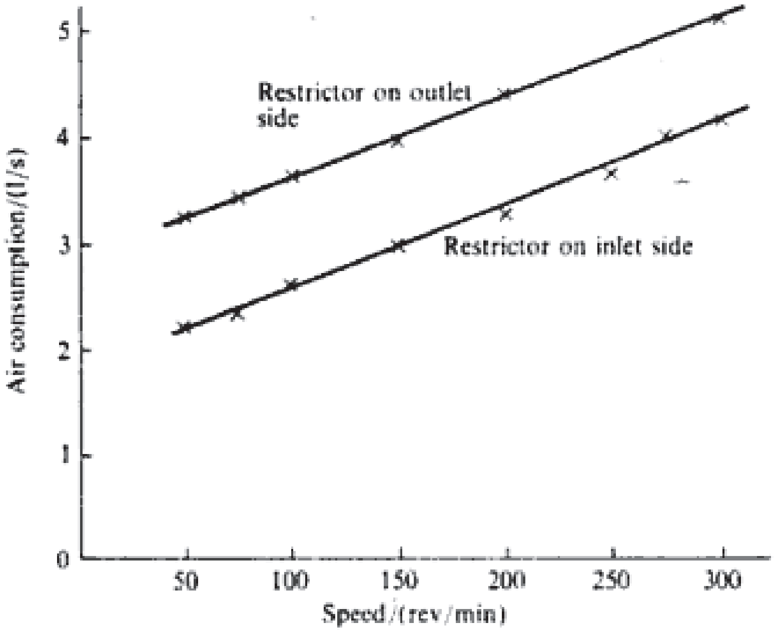


Fig. 12.30 Test results for a vane-type air motor with restrictor control (no load)

Problems

12.1 Air is to be compressed in a single-stage reciprocating compressor from 1.013 bar and 15 °C to 7 bar. Calculate the indicated power required for a free air delivery of 0.3 m^3/min, when the compression process is as follows:
(i) isentropic;
(ii) reversible isothermal;
(iii) polytropic, with $n = 1.25$.
What will be the delivery temperature in each case?
(1.31 kW; 0.98 kW; 1.20 kW; 227.3 °C; 15 °C; 150.9 °C)

12.2 The compressor of Problem 12.1 is to run at 1000 rev/min. If the compressor is single-acting and has a stroke/bore ratio of 1.2/1, calculate the bore size required.
(68.3 mm)

12.3 A single-stage, single-acting air compressor running at 1000 rev/min delivers air at 25 bar. For this purpose the induction and free air conditions can be taken as 1.013 bar and 15 °C, and the FAD as 0.25 m^3/min. The clearance volume is 3% of the swept volume and the stroke/bore ratio is 1.2/1. Calculate:
(i) the bore and stroke;
(ii) the volumetric efficiency;
(iii) the indicated power;
(iv) the isothermal efficiency.
Take the index of compression and re-expansion as 1.3.
(73.2 mm; 87.8 mm; 67.7%; 2 kW; 67.7%)

12.4 The compressor of Problem 12.3 has actual induction conditions of 1 bar and 40 °C, and the delivery pressure is 25 bar. Taking the bore and stroke as calculated in Problem 12.3, calculate the FAD referred to 1.013 bar and 15 °C and the indicated power required. Calculate also the volumetric efficiency and compare it with that of Problem 12.3.
(0.226 m^3/min; 1.98 kW; 61.2%; 67.7%)

12.5 A single-acting compressor is required to deliver air at 70 bar from an induction pressure of 1 bar, at the rate of 2.4 m^3/min measured at free air conditions of 1.013 bar and 15 °C. The compression is carried out in two stages with an ideal intermediate pressure and complete intercooling. The clearance volume is 3% of the swept volume in each cylinder and the compressor speed is 750 rev/min. The index of compression and re-expansion is 1.25 for both cylinders and the temperature at the end of the induction stroke in each cylinder is 32 °C. The mechanical efficiency of the compressor is 85%. Calculate:

(i) the indicated power required;
(ii) the saving in power over single-stage compression between the same pressures;
(iii) the swept volume of each cylinder;
(iv) the required power output of the drive motor.

(22.74 kW; 5.98 kW; 0.00396 m^3, 0.000474 m^3; 26.75 kW)

12.6 For the compressor of Problem 12.5 calculate the heat rejected per minute to the jacket cooling water of each stage, and the heat rejected per minute to the intercooler. Assume that 50% of the friction power in each stage is transferred to the jacket cooling water.

(264 kJ/min; 478 kJ/min)

12.7 A single-cylinder, single-acting air compressor of 200 mm bore by 250 mm stroke is constructed so that its clearance can be altered by moving the cylinder head, the stroke being unaffected.

(a) Using the data below calculate:
(i) the free air delivery;
(ii) the power required from the drive motor.

Data Clearance volume set at 700 cm^3; rotational speed, 300 rev/min; delivery pressure, 5 bar; suction pressure and temperature, 1 bar and 32 °C; free air conditions, 1.013 bar and 15 °C; index of compression and re-expansion, 1.25; mechanical efficiency, 80%.

(b) To what minimum value can the clearance volume be reduced when the delivery pressure is 4.2 bar, assuming that the same driving power is available and that the suction conditions, speed, value of index, and mechanical efficiency, remain unaltered?

(1.68 m^3/min; 7.1 kW; 458 cm^3)

12.8 A single-acting, single-cylinder air compressor running at 300 rev/min is driven by an electric motor. Using the data given below, and assuming that the bore is equal to the stroke, calculate:

(i) the free air delivery;
(ii) the volumetric efficiency;
(iii) the bore and stroke.

Data Air inlet conditions, 1.013 bar and 15 °C; delivery pressure, 8 bar; clearance volume, 7% of swept volume; index of compression and re-expansion, 1.3; mechanical efficiency of the drive between motor and compressor, 87%; motor power output, 23 kW.

(4.47 m^3/min; 72.7%; 297 mm)

12.9 A two-stage air compressor consists of three cylinders having the same bore and stroke. The delivery pressure is 7 bar and the FAD is 4.2 m^3/min. Air is drawn in at 1.013 bar, 15 °C and an intercooler cools the air to 38 °C. The index of compression is 1.3 for all three cylinders. Neglecting clearance calculate:

(i) the intermediate pressure;
(ii) the power required to drive the compressor;
(iii) the isothermal efficiency.

(2.19 bar; 16.2 kW; 84.5%)

12.10 A four-stage compressor works between limits of 1 bar and 112 bar. The index of compression in each stage is 1.28, the temperature at the start of compression in each

stage is 32 °C, and the intermediate pressures are so chosen that the work is divided equally among the stages. Neglecting clearance, calculate:
(i) the temperature at delivery from each stage;
(ii) the volume of free air delivered per kilowatt-hour at 1.013 bar and 15 °C;
(iii) the isothermal efficiency.

(122 °C; 6.23 m^3/kW h; 87.6%)

12.11 A single-cylinder, single-acting reciprocating air compressor supplies a water-cooled receiver from which the air is drawn off for process work. Taking the polytropic index of compression and re-expansion as 1.3, and using the data below, calculate:
(i) the pressure in the receiver;
(ii) the rate of heat rejection from the receiver;
(iii) the volumetric efficiency of the compressor;
(iv) the required power input to the compressor.

Data Cylinder bore, 200 mm; stroke, 250 mm; rotational speed, 440 rev/min; clearance volume, 5% of swept volume; ambient pressure and temperature in compressor house, 1.01 bar and 10 °C; average pressure and temperature during the induction stroke, 1 bar and 20 °C; volume flow rate of air drawn off for process work, 0.6 m^3/min at 17 °C.

Note: Use a trial-and-error method for part (i).

(5 bar; 8.14 kW; 83.9%; 9.85 kW)

12.12 Air at 1.013 bar and 15 °C is to be compressed at the rate of 5.6 m^3/min to 1.75 bar. Two machines are considered: (a) the Roots blower; and (b) a sliding vane rotary compressor. Compare the powers required, assuming for the vane type that internal compression takes place through 75% of the pressure rise before delivery takes place, and that the compressor is an ideal uncooled machine.

(6.88 kW; 5.71 kW)

12.13 Air is compressed in a two-stage vane-type compressor from 1.013 bar to 8.75 bar. Using the data below, and assuming equal pressure ratios in each stage, that compression is complete in each stage, that the machine operates in an ideal manner, and is uncooled apart from the intercooler, calculate:
(i) the power required;
(ii) the volume flow rate measured at the delivery pressure.

Data Free air delivery, 42 m^3/min at 1.013 bar and 15 °C; intercooling between stages is 75% complete.

(187 kW; 7.21 m^3/min)

12.14 The following particulars refer to a single-acting air motor: cylinder diameter 380 mm; stroke 610 mm; speed 200 rev/min; supply pressure and temperature 6.2 bar and 150 °C; back pressure 1.03 bar; index of expansion and compression 1.35; cut-off ratio 0.46; clearance volume 20% of swept volume; mechanical efficiency 95%.

Assuming that the temperature and pressure of the air in the clearance space at the beginning of admission are 6.2 bar and 150 °C, calculate:
(i) the air consumption;
(ii) the air temperature after blow-down;
(iii) the fraction of stroke travelled by the piston before recompression begins;
(iv) the shaft power developed.

(0.54 kg/s; −14.3 °C; 0.463; 72.9 kW)

Reference

12.1 BS 1571 *Testing of Positive Displacement Compressors and Exhausters* Part I (1987), Part II (1984).