UTM UNIVERSITI TEKNOLOGI MALAYSIA OPENCOURSEWARE

Thermodynamics II
Chapter 3
Compressors

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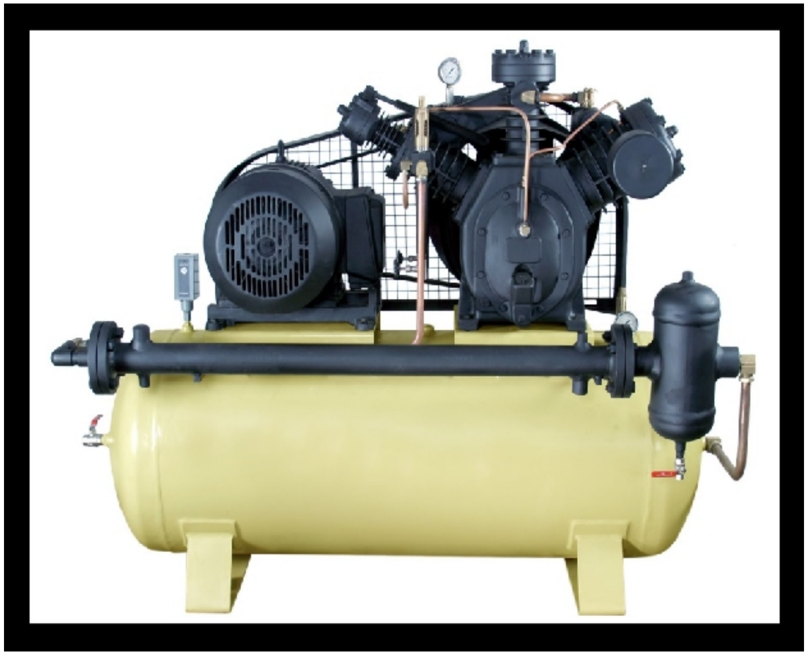
Coverage

- Introduction
- Indicated Work, Mechanical Efficiency
- Condition for Minimum Work
- Isothermal Efficiency
- Compressors with Clearance
- Volumetric Efficiency, Free Air Delivery
- Multistage Compression
- Ideal Intermediate Pressure

Introduction

- Compressed air is air kept under a pressure that is greater than atmospheric pressure.



- In industry, compressed air is so widely used that it is often regarded as the fourth utility, after electricity, natural gas and water.

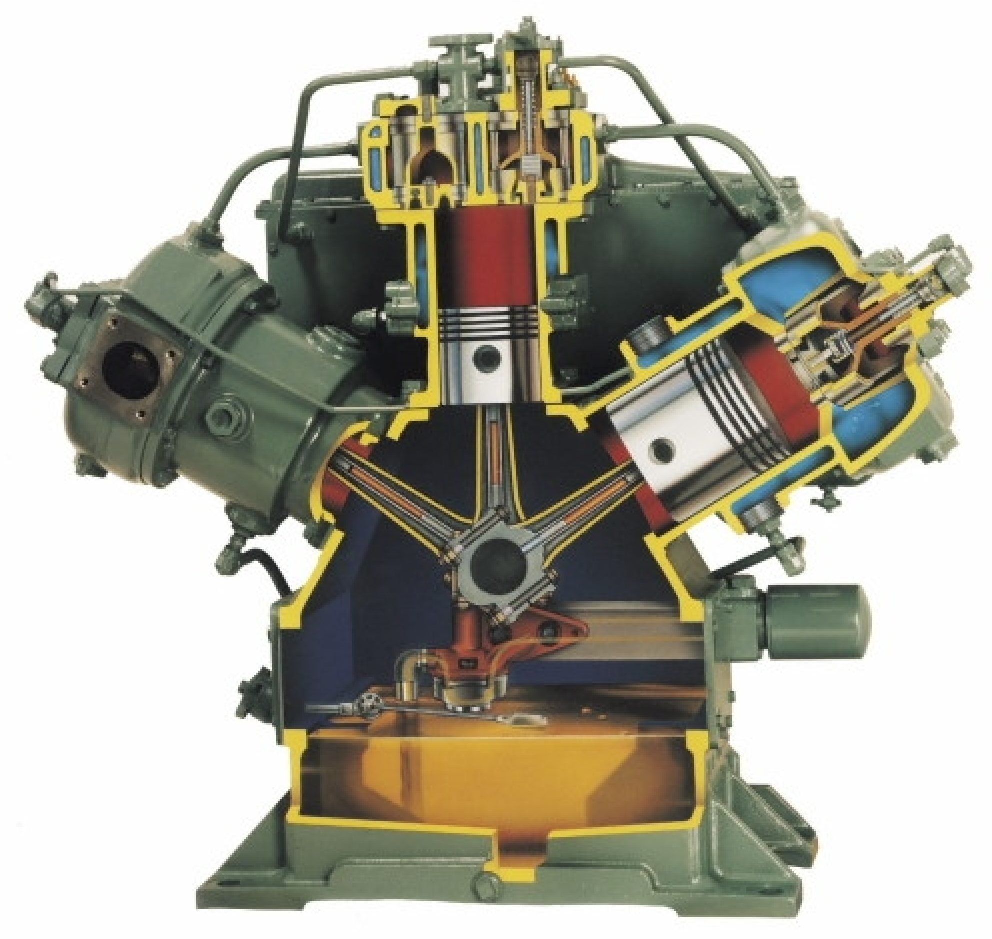
Usages

Compressed air is used for many purposes, including:

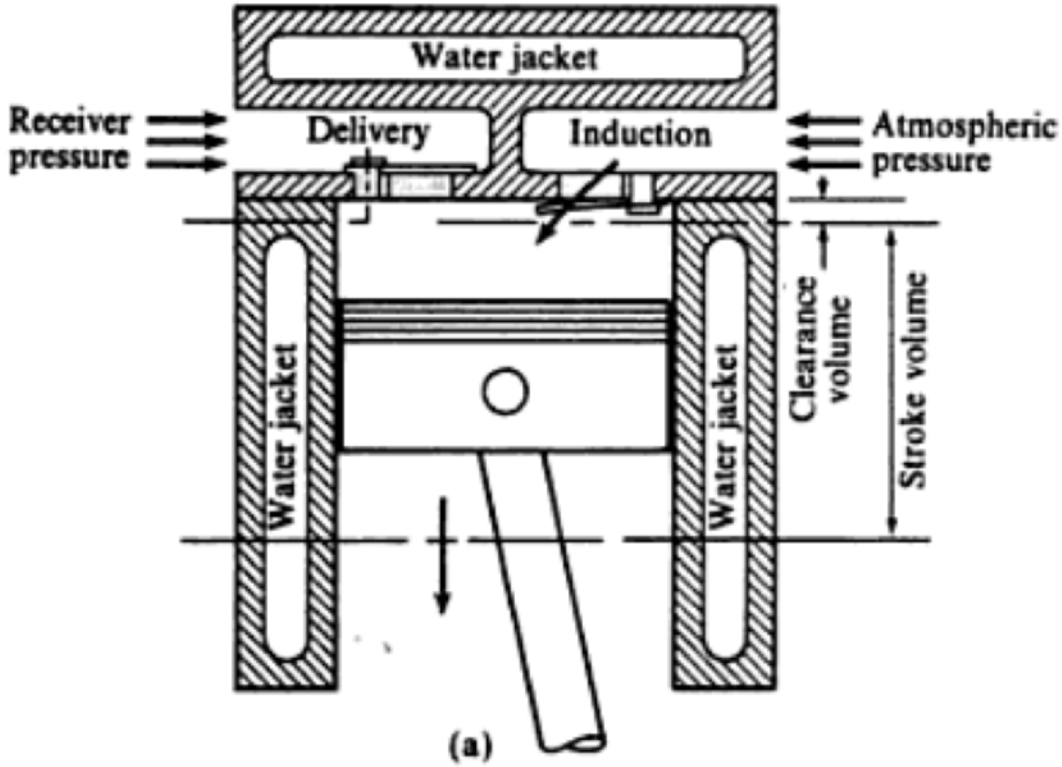
- Pneumatics, the use of pressurized gases to do work
    - Pneumatic post, using capsules to move paper and small goods through tubes.
    - Air tools
    - HVAC control systems
- Vehicle propulsion (compressed air vehicle)
- Energy storage (compressed air energy storage)
- Air brakes, including:
    - railway braking systems
    - road vehicle braking systems
- Scuba diving, for breathing and to inflate buoyancy devices
- Refrigeration using a vortex tube
- Gas dusters for cleaning electronic components that cannot be cleaned with water
- Air-start systems in engines
- Ammunition propulsion in:
    - Air guns, Airsoft equipment, Paintball equipment

Compressor types

- Positive Displacement Machines
  (high pressure ratio, low mass flow rates)
  - Rotating
    - Screw compressors (Lysholm)
    - Scroll compressor
    - Roots blowers
  - Alternating (Reciprocating Compressor)
- Turbocompressors
  (low pressure ratio, high mass flow rates)
  - Centrifugal compressor
  - Axial compressor
  - Mixed-flow compressor

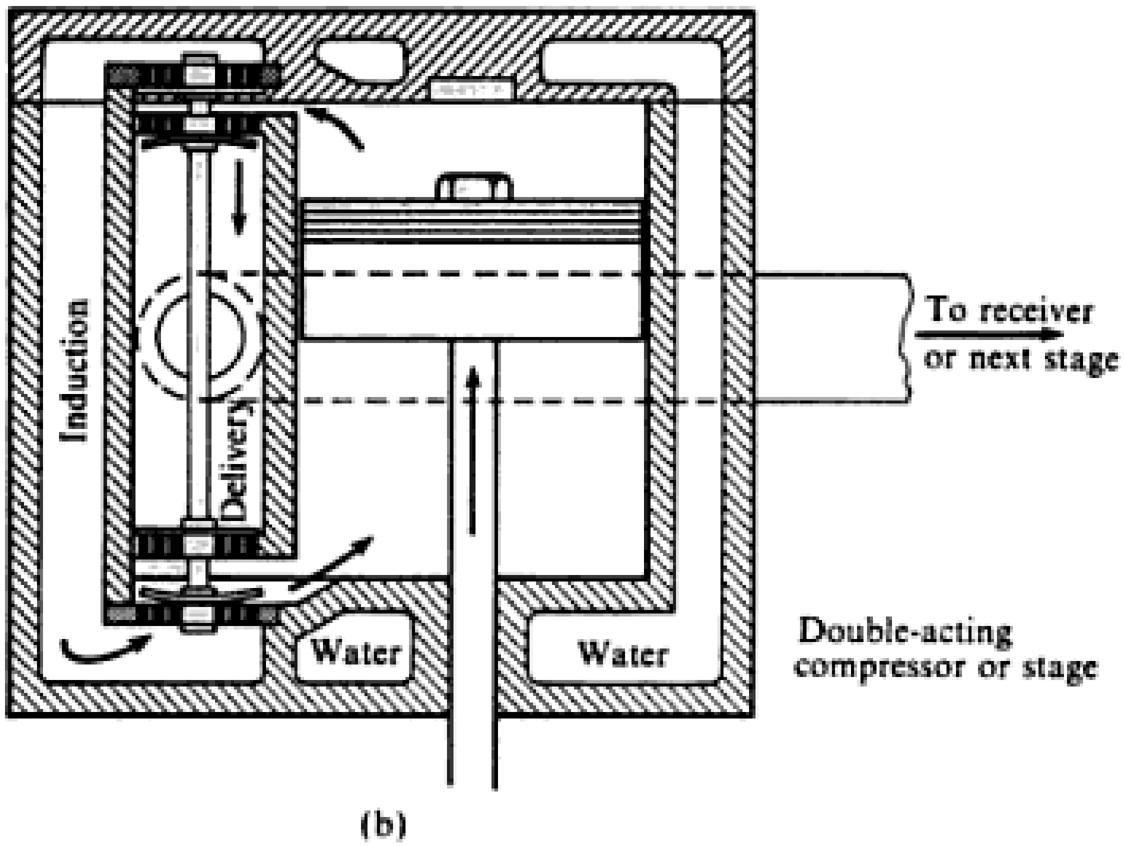


Reciprocating Compressor



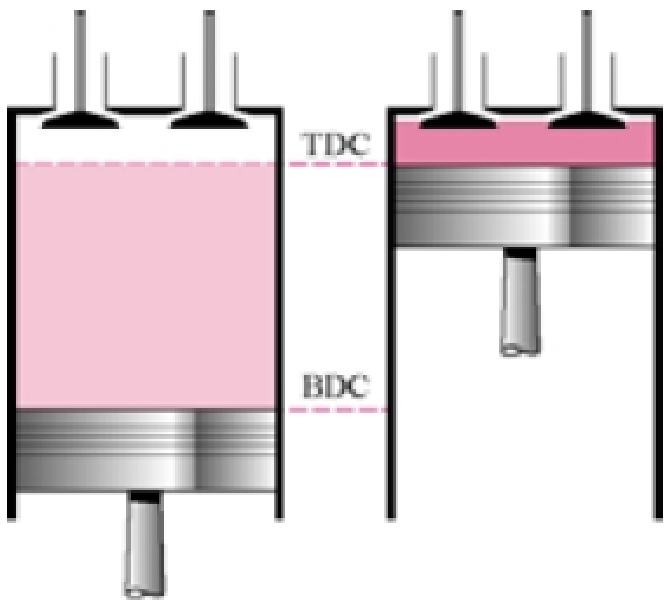
Single Acting

Reciprocating Compressor



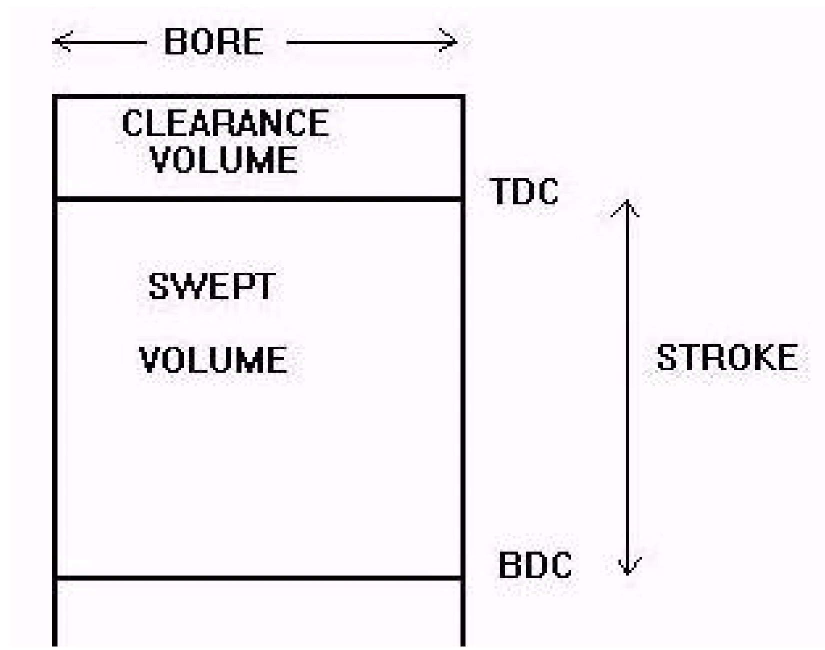
Double Acting

Piston-cylinder terminologies



(*a*) Displacement volume

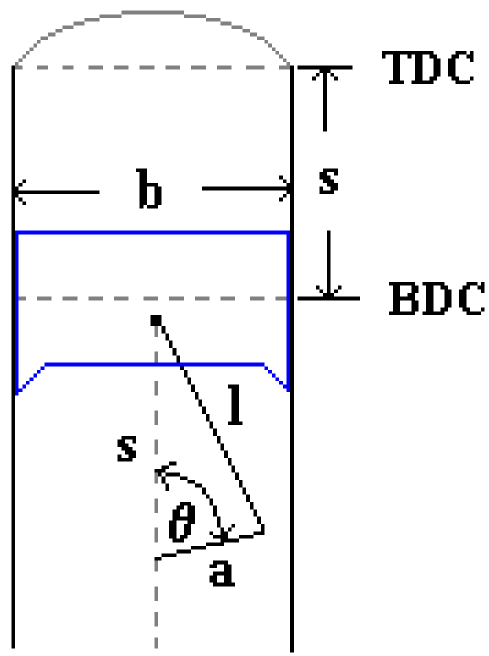
(*b*) Clearance volume



TDC – Top Dead Center

BDC – Bottom Dead Center

Piston-cylinder terminologies



b – Bore, Diameter

s – Stroke

l – Connecting Rod Length

a – Crank Throw = ½ stroke

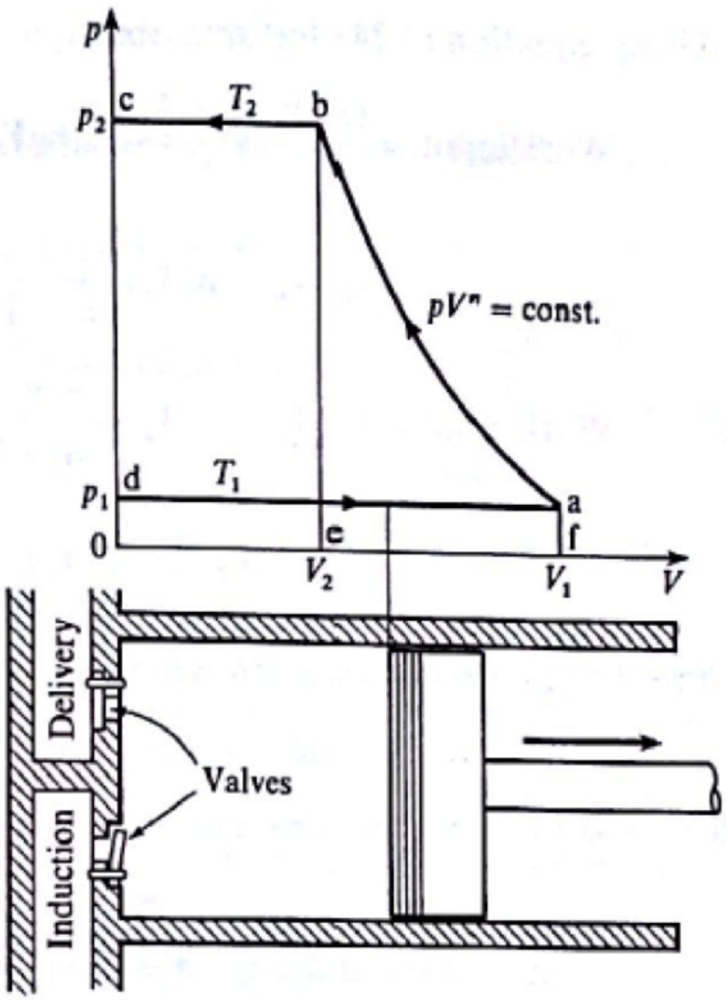


Fig. 12.2 Pressure–volume diagram for a reciprocating compressor with clearance neglected

$$\frac{P_2}{P_1} = \text{Pressure Ratio} = r_P$$

Compressor Operation

- Process d – a : Intake or Induction
  - Piston moves from TDC to BDC
  - Intake valve opens and air induced into cylinder
  - Pressure P_1 and temperature T_1 remain constant.
- Process a – b : Compression
  - Intake valve closes and piston moves towards TDC
  - Compression follows the polytropic process $Pv^n=c$ until P_2 is reached.

Compressor Operation

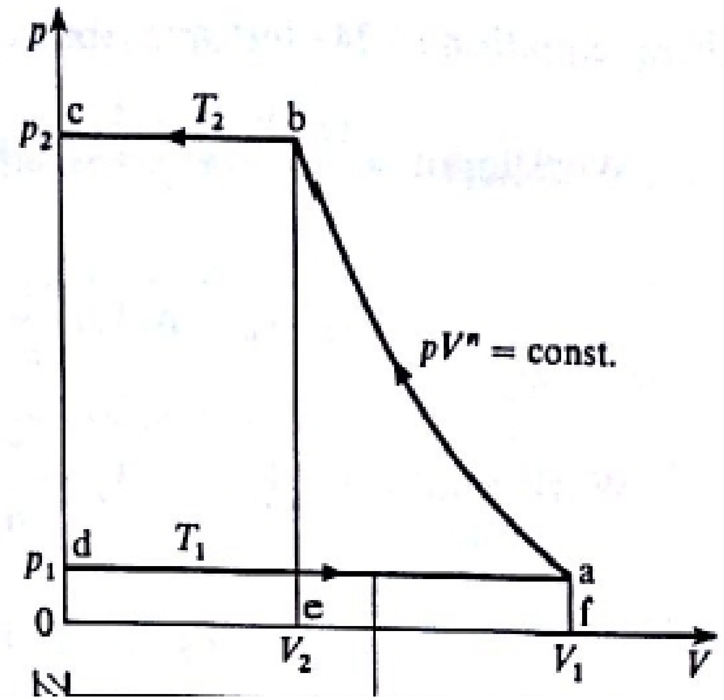
- Process b – c : Delivery
  - Delivery valve opens
  - Compressed air exits and delivered.
  - Pressure P_2 and temperature T_2 remain constant.
- Process c – d : Expansion
  - Both valves remain closed as the cycle returns to the initial state
  - Constant volume if without clearance
  - Polytropic expansion if with clearance

Indicated Work

- Indicated by P-v diagram, (P-v diagram = Indicator diagram)

For a cycle

$$W_{ind} = \int_1^2 P\,dV$$
$$= \text{area of abcd}$$
$$= \text{abef} + \text{bcoe} - \text{adof}$$
$$= \frac{P_2V_b - P_1V_a}{n-1} + P_2V_b - P_1V_a$$
$$= \frac{n}{n-1}(P_2V_b - P_1V_a)$$



Recall polytropic relationship between two states

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}} = \left(\frac{v_1}{v_2}\right)^{(n-1)}$$

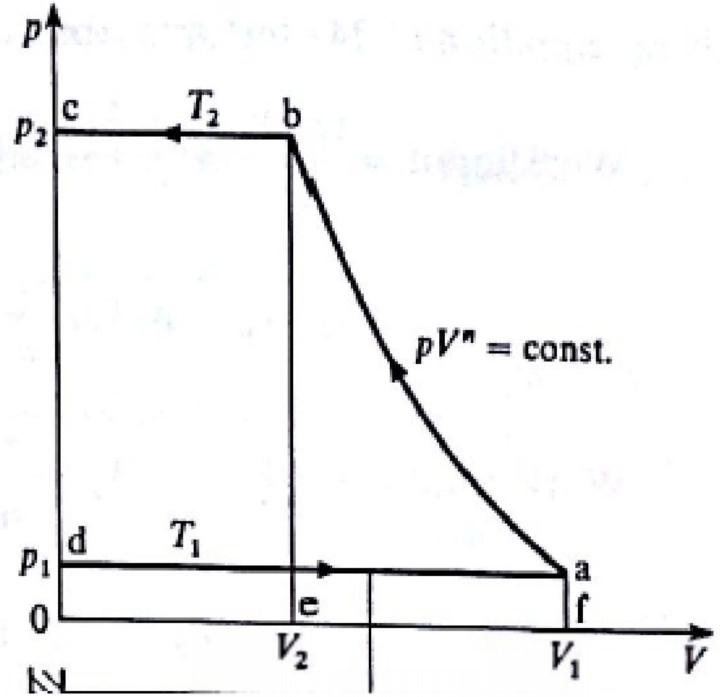
Indicated Work

Can also be considered as open system

$$W_{ind} = \int_{P_1}^{P_2} V dP$$

$$= \text{area of abcd}$$

$$= \frac{n}{n-1}(P_2V_b - P_1V_a)$$



And since PV = mRT

$$W_{ind} = \frac{n}{n-1}(mRT_2 - mRT_1)$$

$$= \frac{n}{n-1}mR(T_2 - T_1)$$

$$= \frac{n}{n-1}mRT_1\left(\frac{T_2}{T_1} - 1\right)$$

$$= \frac{n}{n-1}mRT_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right)$$

$$= \frac{n}{n-1}P_1V_a\left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right)$$

Power (and Rates)

- Has to take into account single or double acting
- W_{ind} is work per cycle of P-v diagram.
- If single acting, one cycle per crank revolution
- If double acting, two cycles per crank revolution (one cycle each for both sides of piston face).
  - Mass flow rate is doubled accordingly.

$$\dot{W} = \frac{n}{n-1}\dot{m}RT_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right)$$

$$= \frac{n}{n-1}P_1\dot{V}\left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right)$$

$$\dot{m} = m \times N \times action$$

Mechanical Efficiency

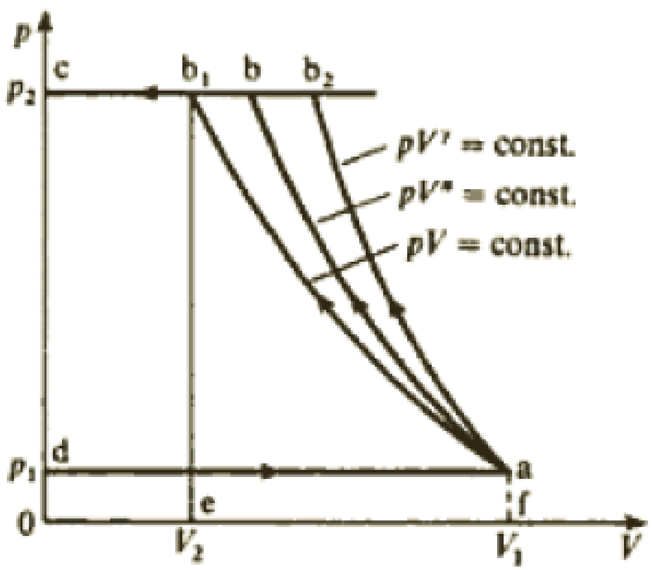
- The actual power input into the compressor is larger than the indicated power, to overcome friction and other losses.

Shaft power = Indicated power + Friction power loss

$$\text{Mechanical Efficiency} = \frac{\text{Indicated power}}{\text{shaft power}}$$

Other losses can also be taken into account accordingly

Condition for Minimum Work



- We aim to reduce the input work
- d-a is the stroke, determined by cylinder design and measurement
- P_2 is desired delivery pressure. As long as P_2 is reached, the compressor has done its job.

- Only the compression process can be adjusted by varying n, the polytropic index.
- Isothermal process (n=1) results in minimum work (smallest area).
- Compressors are cooled by water jackets or cooling fins

Isothermal Work, Isothermal Efficiency

- Integrating by isothermal process, Pv=c

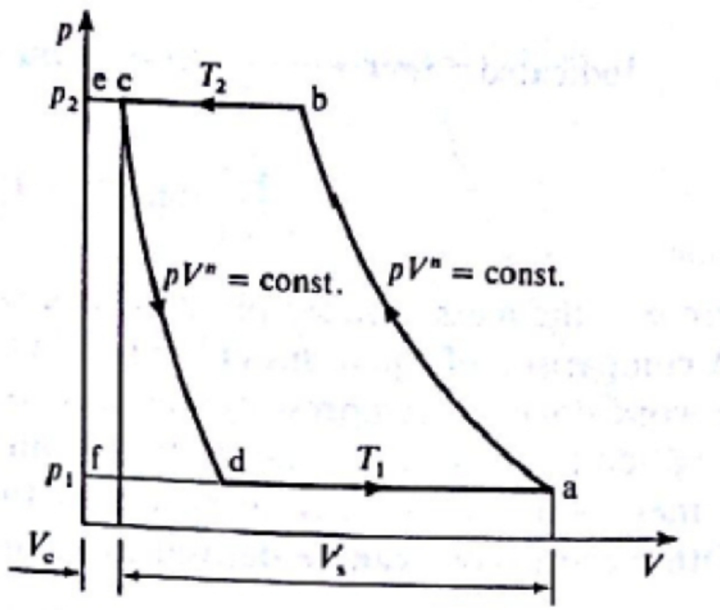
$$W_{\text{isothermal}} = P_1 V_a \ln\left(\frac{P_2}{P_1}\right)$$

$$= mRT \ln\left(\frac{P_2}{P_1}\right)$$

- Isothermal efficiency

$$\eta_{\text{isothermal}} = \frac{\text{Isothermal Work}}{\text{Indicated Work}}$$

Compressors with Clearance



- Clearance is needed for free movements of piston and valves
- Clearance volume is V_c.
- When delivery is completed (b-c), there is still compressed air at P_2 and T_2 in the clearance volume.
- When intake stroke begins at V_c, no outside air can enter yet until the residual compressed air has expanded down to P_1 and T_1.
- Thus, *having clearance reduces the volume of inducted air* from (V_a-V_c) originally to only (V_a-V_d)

Compressors with Clearance

- Mass of air, $m_a = m_b$, and $m_d = m_c$
- The amount of air handled, $m = m_a - m_d = m_b - m_c$
- W_{ind} = area abcd
  = area abef – area cefd

$$W_{ind} = \frac{n}{n-1} m_a R(T_2 - T_1) - \frac{n}{n-1} m_d R(T_2 - T_1)$$

$$= \frac{n}{n-1}(m_a - m_d)R(T_2 - T_1)$$

$$= \frac{n}{n-1} mR(T_2 - T_1)$$

Even though Work depends on clearance, but *work per unit mass* does not depend on it.

Free Air Delivery, FAD

- FAD is the amount of air handled (delivered) by the compressor.
- FAD is given as the volumetric flow rate of air (measured at free air conditions P_o and T_o)

$$\text{FAD} = \dot{V}_u = \frac{\dot{m}RT_o}{P_o}$$

Actually, this is easier given by the mass flow rate since it does not depend on P and T

Volumetric Efficiency

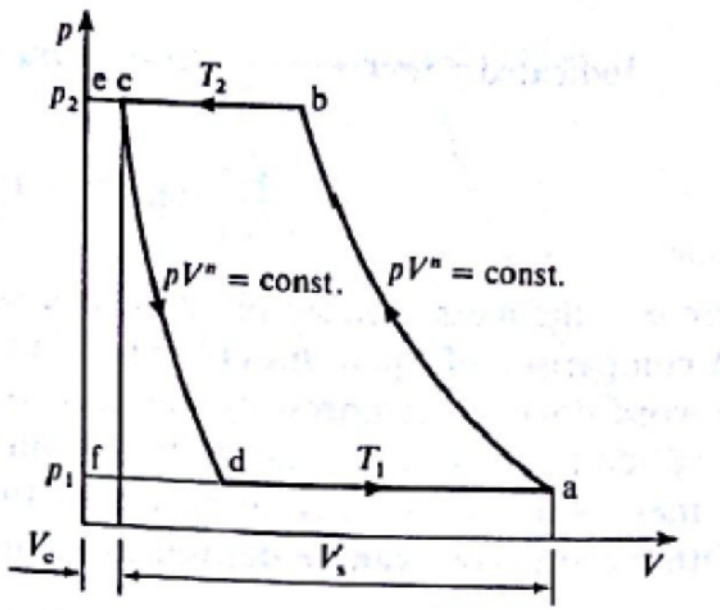
$$\eta_v = \frac{m_u}{m_s}$$

$$\frac{\text{The mass of gas entering}}{\text{The mass of gas that should fill the swept volume at the same reference condition (free air condition)}}$$

$$\eta_v = \frac{V_u}{V_s}$$

$$\frac{\text{The volume of gas entering measured at free air condition}}{\text{The swept volume of cylinder}}$$

Volumetric Efficiency



$$\eta_v = \frac{V_u}{V_s} = \frac{(V_a - V_d)}{(V_a - V_c)}$$

$$\eta_v = 1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{1/n} - 1\right]$$

- The result above is assuming that the in-cylinder condition (T_1, P_1) is the same as free air condition (T_o, P_o)

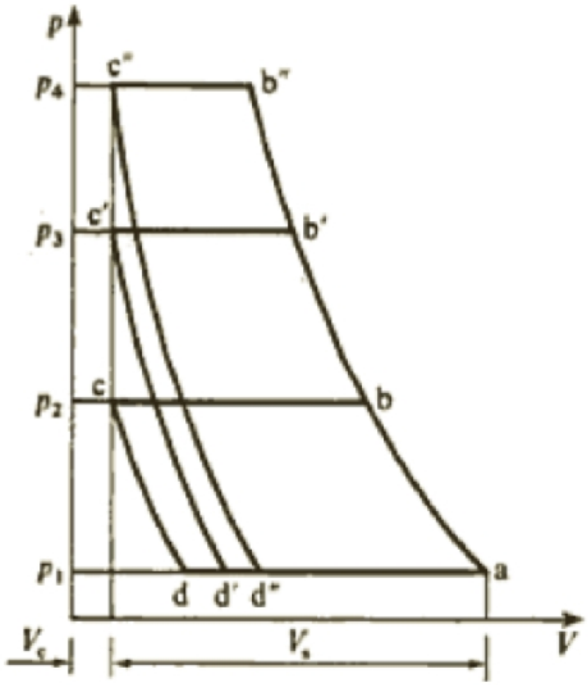
Volumetric Efficiency

- The entering air is actually being heated by the hot cylinder walls and there has to be a pressure difference ($P_o - P_1$) so that air can flow into the cylinder.
- We can use the unchanging mass to get the correction factor to account for these differences

$$V_u = (V_a - V_d)\frac{T_o}{T_1}\frac{P_1}{P_o}$$

$$\eta_v = \left\{1 - \frac{V_c}{V_s}\left[\left(\frac{P_2}{P_1}\right)^{1/n} - 1\right]\right\}\frac{T_o}{T_1}\frac{P_1}{P_o}$$

Multistage Compression



- For a given V_s, increasing r_p will
  - decrease η_v.
  - Increase delivery temperature
- To achieve high pressures while avoiding those problems
  - Do Multistage Compression

- At some intermediate pressure P_i, the gas is sent to a smaller cylinder to be compressed further.
- This also allows us to cool the gas (intercooling) to reduce compression work.

Multistage Compression

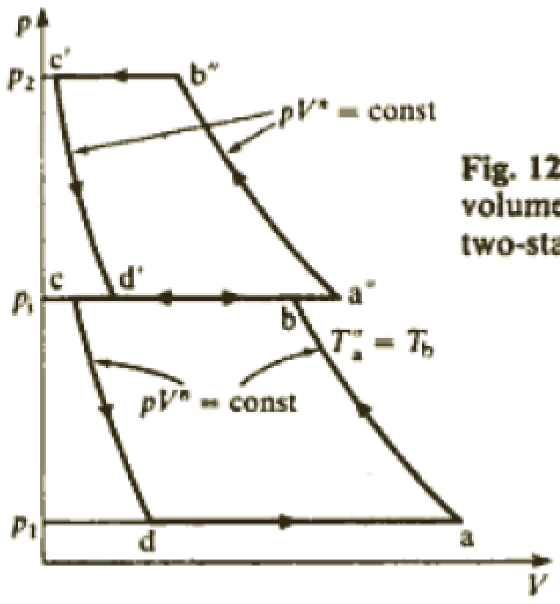


Fig. 12.13 Pressure–volume diagram for two-stage compression

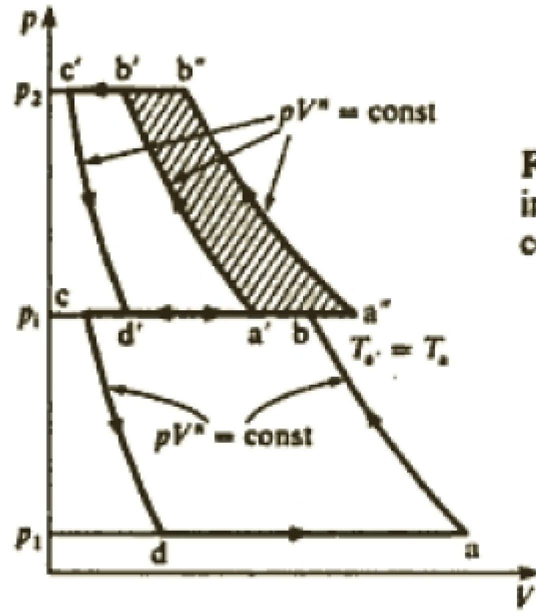
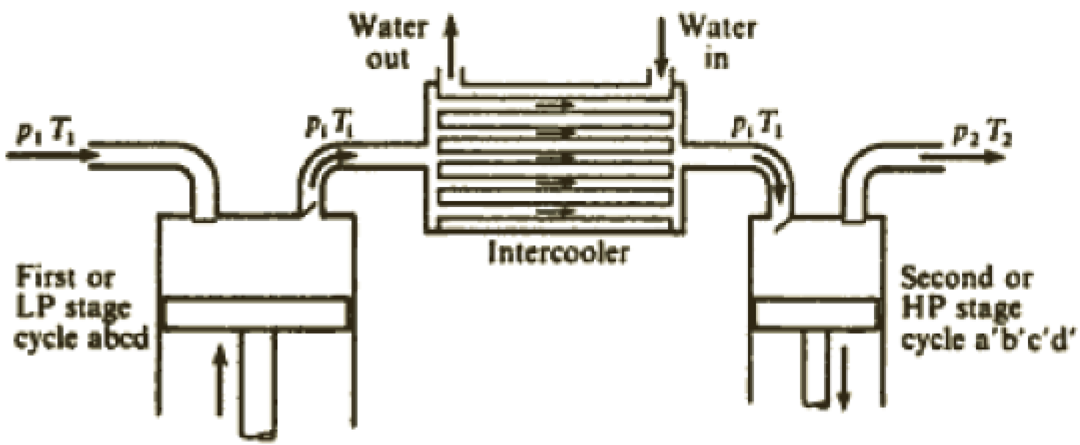


Fig. 12.14 Effect of intercooling on the compression work



Multistage Compression

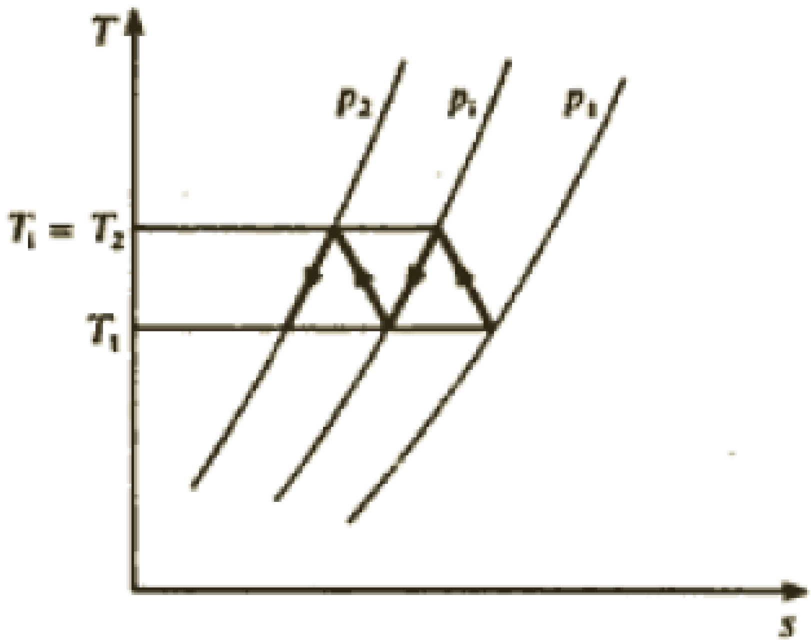


Fig. 12.16 *T–s* diagram showing intercooling and aftercooling

- Complete Intercooling if
  - Intermediate temperature T_i is cooled back to the same temperature as T_1.

Optimum Intermediate Pressure

- The chosen P_i affects the amount of compression work that has to be supplied.
- An optimum P_i will give us the minimum compressor work.
- Let's assume complete intercooling.

$W_{total} = W_{Low\ Stage} + W_{High\ Stage}$

$$W_{Total} = \frac{n}{n-1} mRT_1 \left(\left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}} - 1 \right) + \frac{n}{n-1} mRT_i \left(\left(\frac{P_2}{P_i} \right)^{\frac{n-1}{n}} - 1 \right)$$

Since $T_i = T_1$,

$$W_{Total} = \frac{n}{n-1} mRT_1 \left(\left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_i} \right)^{\frac{n-1}{n}} - 2 \right)$$

Optimum Intermediate Pressure

- For a fixed P_1, T_1 and P_2, we can the optimum P_i that gives us minimum W_{total} by

$$\frac{dW_{Total}}{dP_i} = 0$$

$$\frac{dW_{Total}}{dP_i} = \frac{d}{dP_i}\left(\frac{n}{n-1}mRT_1\left(\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}} - 2\right)\right) = 0$$

$$= \frac{d}{dP_i}\left(\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}} - 2\right) = 0$$

$$P_i^{\,2} = P_2 P_1$$

$$P_{i-optimum} = \sqrt[2]{P_2 P_1}$$

$$\frac{P_i}{P_1} = \frac{P_2}{P_i} = r_{p-optimum} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{2}}$$

Optimum Intermediate Pressure

- So, for minimum compressor work
  - Complete intercooling
  - Same pressure ratio for all stages
- This can be generalized to more than two stages

$$W_{Total} = \frac{n}{n-1} mRT_1 \left(\left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}} - 1 \right) + \frac{n}{n-1} mRT_i \left(\left(\frac{P_2}{P_i} \right)^{\frac{n-1}{n}} - 1 \right)$$

$$W_{Total} = \frac{2n}{n-1} mRT_1 \left(\left(\frac{P_i}{P_1} \right)^{\frac{n-1}{n}} - 1 \right)$$

$$W_{Total} = \frac{2n}{n-1} mRT_1 \left(\left(\frac{P_2}{P_1} \right)^{\frac{n-1}{2n}} - 1 \right)$$

Optimum Intermediate Pressure

- This can be generalized to more than two stages (z = number of stages, P1 = intake pressure, P2 = final pressure)
- For minimum compressor work
  - Complete intercooling
  - Same pressure ratio for all stages

$$r_{p-optimum} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{z}}$$

$$W_{Total} = \frac{zn}{n-1} mRT_1 \left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{zn}} - 1\right)$$