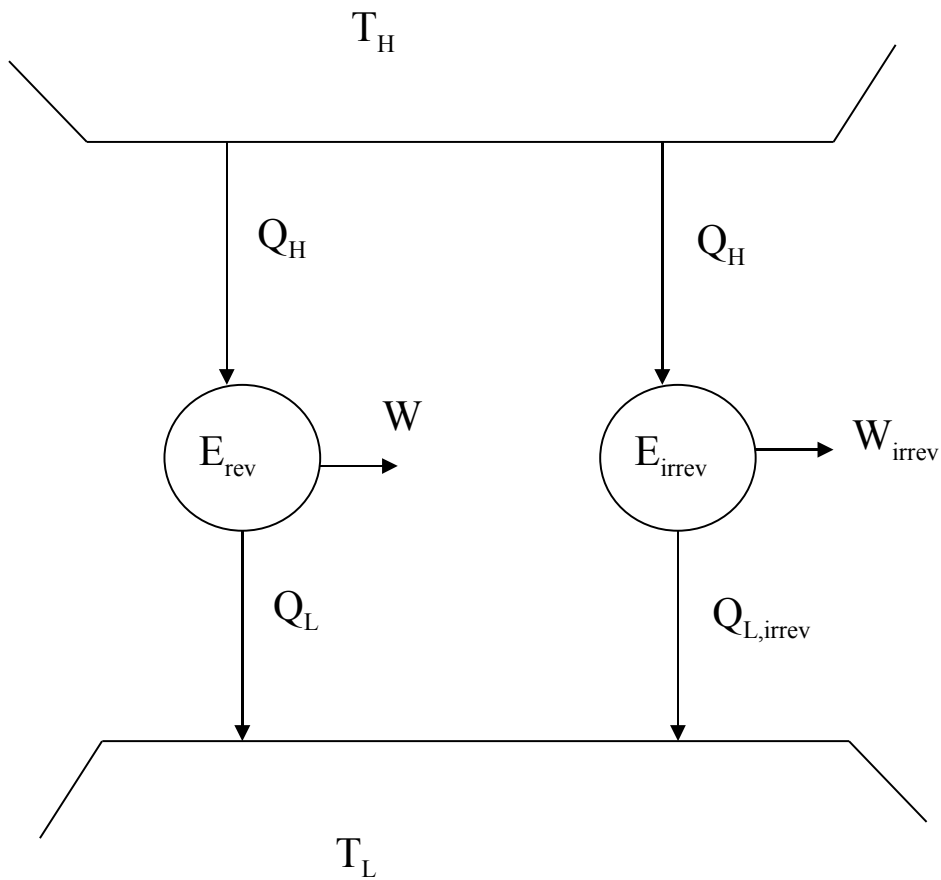


Clausius Inequality



Consider two heat engines between the same emperatures

Consider $\frac{\delta Q}{T}$ of the whole cycle

(a) E_{rev}

$$\begin{aligned}\oint \left(\frac{\delta Q}{T} \right)_{\text{rev}} &= \int \frac{\delta Q_H}{T_H} - \int \frac{\delta Q_L}{T_L} \\ &= \frac{1}{T_H} \int \delta Q_H - \frac{1}{T_L} \int \delta Q_L \\ &= \frac{Q_H}{T_H} - \frac{Q_L}{T_L} \\ &= 0\end{aligned}$$

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{rev}} = 0$$

(b) E_{irrev}

$$Q_{L,\text{irrev}} > Q_L$$

$$Q_{L,\text{irrev}} = Q_L + \Delta Q$$

$$\begin{aligned}\oint \left(\frac{\delta Q}{T} \right)_{\text{irrev}} &= \frac{Q_H}{T_H} - \frac{Q_{L,\text{irrev}}}{T_L} \\ &= \frac{Q_H}{T_H} - \frac{Q_L + \Delta Q}{T_L} \\ &= \underbrace{\frac{Q_H}{T_H} - \frac{Q_L}{T_L}}_{=0} - \frac{\Delta Q}{T_L} \\ &= -\frac{\Delta Q}{T_L} < 0\end{aligned}$$

$$\text{so } \oint \left(\frac{\delta Q}{T} \right)_{\text{irrev}} < 0$$

Combined to be:

$$\oint \left(\frac{\delta Q}{T} \right)_{irrev} \leq 0 \quad \text{Clausius Inequality}$$

$= 0$	if reversible
< 0	if irreversible
> 0	impossible