

1. The deflection y of the top of a sailboat mast is

$$y = \frac{FL^4}{8EI}$$

where F is a uniform side loading (lb/ft), L is the height (ft), E is the modulus of elasticity (lb/ft²), and I is the moment of inertia (ft⁴).

- a. Calculate the deflection y and from the Taylor series, estimate the error in calculating the deflection y given the following data

$$F = 50 \pm 2 \left(\frac{\text{lb}}{\text{ft}} \right), L = 30 \pm 0.1 \text{ ft}, E = 1.5 \times 10^8 \pm 0.01 \times 10^8 \text{ (lb/ft}^2\text{)}, \text{ and } I = 0.06 \text{ ft}^4.$$

- b. From Taylor series, derive the expression for error in y for second order error estimate.

2. The trajectory of an artillery shell is defined by the (x, y) coordinates, as displayed in Fig. Q2. The trajectory can be modeled as

$$y = (\tan\theta_0)x - \frac{g}{2v_0^2\cos^2\theta_0}x^2 + y_0$$

Find the appropriate artillery barrel angle θ_0 , if the initial velocity $v_0 = 500$ m/s and the distance to the enemy target x is 10 km. Note that the shell leaves the artillery muzzle at an elevation of $y_0 = 2.5$ m and the target is at ground level. Express the final result in degrees. Use a value of 9.81 m/s^2 for g . State the method of your choice and your reason. Perform only 6 iterations on a spreadsheet. Copy the spreadsheet results on your answer script.

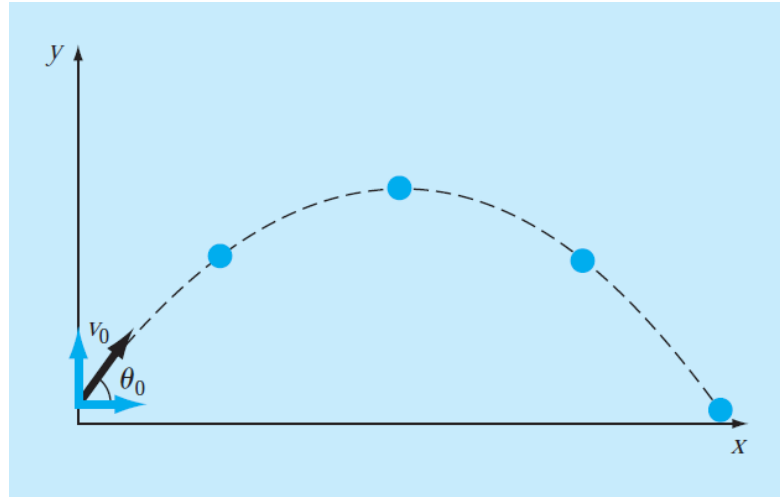


Figure Q2

3. For the following system:

$$\begin{aligned} 10x_1 + 2x_2 - x_3 &= 27 \\ -3x_1 - 6x_2 + 2x_3 &= -61.5 \\ x_1 + x_2 + 5x_3 &= -21.5 \end{aligned}$$

- Perform Gauss elimination manually to solve for x_i
- If you use the LU decomposition method for the problem above, write the lower triangular matrix that you need to obtain the solution. (Do not perform LU decomposition itself).

4. For the matrix below

$$\begin{bmatrix} 2 & -1 & 1 & 3 \\ -1 & 2 & 1 & 2 \\ 3 & 1 & -1 & 2 \\ 1 & -1 & 1 & 2 \end{bmatrix}$$

- Find the characteristic polynomial and state the method that you use to obtain it.
- Write the Matlab commands and output to find the roots of the characteristic polynomial.
- Write the Matlab commands and output to calculate the eigenvalues and eigenvectors of the matrix.