MMJ 1113 COMPUTATIONAL METHODS FOR ENGINEERS — Semester I 2014-2015
Test 2 — 90 minutes

1. **(Short)** From Taylor Series, derive the central difference formula for the first derivative. Show that it is second order accurate.

2. **(Long)** Solve the system of simultaneous ordinary differential equations below over the range $x = 0$ to 1 using a step size of 0.25

$$\frac{dy}{dx} = -2y + 4e^{-x}$$
$$\frac{dz}{dx} = -\frac{yz^2}{3}$$

with initial conditions $y(0) = 2$ and $z(0) = 4$. What is the order of accuracy of your chosen method? Write a short Matlab code to find the answer above using `ode23` along with a user-defined function.

3. **(Long)** From an experiment, the dynamic viscosity of water μ (10^{-3} N.s/m^2) is related to temperature T in the following manner.

T	0	5	20	40
μ	1.787	1.519	1.002	0.6529

(a) Using Table 1 below, and by calculating additional required information, estimate the dynamic viscosity of water at $T = 18^oC$ using second order polynomial regression.

Table 1 : Information for quadratic regression (incomplete)

	T	μ	T^2	T^3	$T.\ \mu$
	0	1.787	0	0	0
	5	1.519	25	125	7.595
	20	1.002	400	8000	20.04
	40	0.6529	1600	64000	26.116
Sum :	65	4.9609	2025	72125	53.751

(b) If the effect of temperature on dynamic viscosity is known to follow Andrade's equation,

$$\mu = De^m \quad \text{where} \quad m = \frac{B}{T + 273.15} \qquad \text{(Eq. 1)}$$

determine the two unknown constants, B and D using regression.

(c) Estimate the dynamic viscosity at $T = 18^oC$ using Eq.1 and calculate the relative error compared to your answer in (a).

4. **(Long)** A cooling fin extends from an engine block as shown in Fig Q4. Assuming that heat flows only in the x-direction, the thermal equilibrium equation leads to

$$kA\frac{d^2T}{dx^2} - h_{conv}P(T - T_\infty) = 0$$

where k = thermal conductivity, A = cross-sectional area, h_{conv} = convection heat transfer coefficient, T_∞ = surrounding temperature, P = perimeter, and T = temperature at x. From a finite difference formulation using central difference, construct a matrix system that can be used to solve for the temperature distribution in the fin (do not solve this matrix). Formulate for a fin of length L, taking five equal intervals of h. The boundary conditions are fixed temperature $T = T_0$ at $x = 0$, and fixed convection heat flux $-k\frac{dT}{dx} = h_{conv}(T - T_\infty)$ at $x = L$. Use a second order accurate formulation for the boundary condition too.

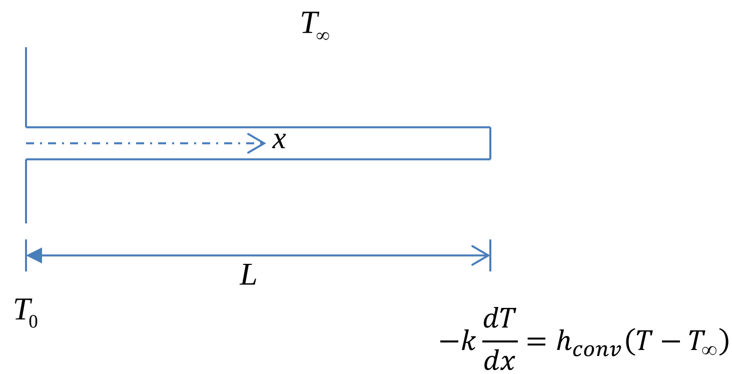

Fig Q4