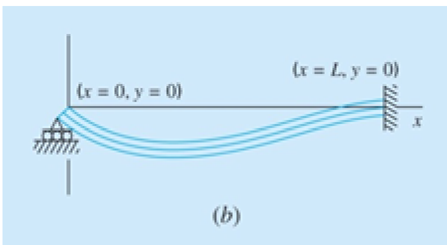
COMPUTATIONAL METHODS FOR ENGINEERS	Semester II 2013-2014
Test 1	90 minutes

1. (a) Consider the function $v(t) = \frac{gm}{c} - e^{-(c/m)t}$. Use a first order error analysis to estimate the error of v at $t = 6$, if $g = 9.8$ and $m = 50$, but $c = 12.5 \pm 1.5$.

   (b) A uniform beam subject to a linearly increasing distributed load results in deflection, y as $y = \frac{w_0}{120EIL}\left(-x^5 + 2L^2x^3 - L^4x\right)$. By inspection considering error propagation, is the deflection, y sensitive to beam length, L or material property, E? State your reasons.



2. In combustion, the dissociation of CO_2 at 2500K results in the following expression for the equilibrium constant K;

$$K = \frac{z}{(1-z)}\left(\frac{z}{2+z}\right)^{1/2}\left(\frac{P}{P_{ref}}\right)^{1/2}$$

   where z is what is called the *progress variable* where $z = 0$ is the beginning and $z = 1$ is the completion of reaction. Given $P_{ref} = 1$ atm and $P = 10$ atm, $K = 0.0363$, find z. State your reason for using the method of your choice. Do only three iterations. What is the error if the answer is 0.0621? Write a short Matlab code to find the answer above using `fzero` along with a user-defined function.

3. A network of one-way streets with traffic flowing in the directions indicated is shown in Figure 2. The flow rates along the streets are measured as the average number of vehicles per hour.

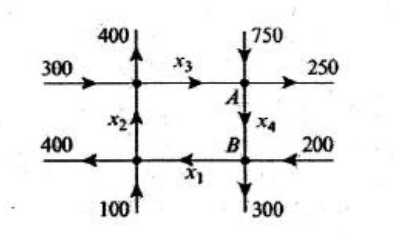
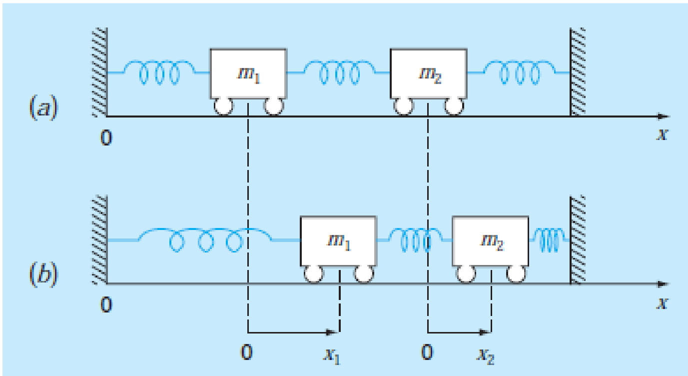


Figure 2: Network of one-way streets.

   a. Set up a system of linear equations whose solution provides the unknown flow rates x_1, x_2, x_3 and x_4.
   b. Solve the system, by three iterations of the Gauss-Seidel method, for the unknown flow rates, and show a Matlab session to solve the same.

4. Figure 3 shows a two-mass, m_1 and m_2, three springs, with k_1, k_2 and k_3 spring constants, system. The equations of motion for the system based on $\sum F = ma$ are given as

$$-k_1x_1 + k_2(x_2 - x_1) = m_1\ddot{x}_1$$
$$-k_2(x_2 - x_1) - k_3x_2 = m_2\ddot{x}_2$$



a. If the solution for the differential equations are $x_1 = A_1\sin(\omega t)$, and $x_2 = A_2\sin(\omega t)$, show that the characteristic equation is given by

$$\left(\frac{k_1 + k_2}{m_1} - \lambda\right)\left(\frac{k_2 + k_3}{m_2} - \lambda\right) - \frac{k_2^2}{m_1 m_2} = 0$$

where λ is the eigenvalue and $\lambda = \omega^2$.

b. Using Power Method, determine the largest eigenvalue and its corresponding eigenvector. Take $k_1 = k_3 = 150$ N/m, $k_2 = 250$ N/m and $m_1 = m_2 = 40$ kg. You may use 10, -11 as your initial guesses. What are your results and error after three iterations?