UNIVERSITI TEKNOLOGI MALAYSIA
FACULTY OF MECHANICAL ENGINEERING

SME 3023 Applied Numerical Methods

Tutorial **Differential Equations**

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1. For the following ordinary differential equation
$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 0 \tag{E1}$$
   (a) find its total solution (manually) subject to the initial conditions $y(0) = 3$ and $y'(0) = 4$ where $y' = dy/dt$,

   (b) use Matlab's `dsolve` to check your solution and write a Matlab script to plot the function $y = f(t)$ representing the solution.

2. The flow through porous media can be described by the Laplace equation
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \tag{E1}$$
   where h is head. Use numerical methods to determine the distribution of head for the system shown in Figure 1.

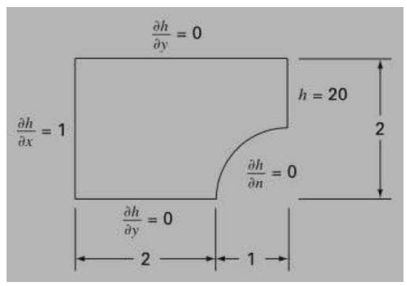

Figure 1: Flow through porous media.

3. A thin rod rod is insulated at all point along its length but exposed at both its end, with one end hotter than the other. The heat conduction equation, taking into consideration the amount of heat stored in the element over a time period, is
$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \tag{E2}$$
   where k is the coefficient of thermal diffusivity, T temperature, x distance along the length of the rod and t is time.

   (a) In an explicit method of solution, Eq. E2 requires approximations for the second derivative in space and the first derivative in time. Using a centred finite-divided difference to represent the former and a forward finite-divided difference to approximate the time derivative, solve for temperature distribution along a 10 cm rod. Take $\Delta x = 2$ cm, $\Delta t = 0.1$ s and $k = 0.020875$. At $t = 0$, the temperature of the rod is zero and the boundary conditions are fixed for all time at $T(x = 0) = 100°$C and $T(x = 10) = 50°$C.

   (b) In a simple implicit method approximated at an advanced time level $l + 1$, Eq. E2 may be written as
$$k\frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{(\Delta x)^2} = \frac{T_i^{l+1} - T_i^l}{\Delta t}$$
   which can be simplified into
$$-\lambda T_{i-1}^{l+1} + (1 + 2\lambda)T_i^{l+1} - \lambda T_{i+1}^{l+1} = T_i^l$$
   where $\lambda = k\Delta t/(\Delta x)^2$. Solve the same problem as in part (a) using this simple implicit method and compare the results.