UNIVERSITI TEKNOLOGI MALAYSIA
FACULTY OF MECHANICAL ENGINEERING

SME 3023 Applied Numerical Methods

Tutorial 2 **Solution of Nonlinear Equations**

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1. Find, without programming, the root of the equation

$$f(x) = \sqrt{4x^2 - 0.04} + 0.24 \sin^{-1}\left(\frac{0.1}{x}\right) - 10.055752 = 0$$

using bisection method with $a = 2$, $b = 10$, and $\epsilon = 0.001$

2. The natural frequency of radial vibration of an elastic sphere, ω, are given by

$$\frac{\tan(ka)}{ka} = \frac{4G}{4G - (\lambda + 2G)(ka)^2} \tag{E1}$$

where the Lame's constant, λ, and the shear modulus, G, are given

$$\lambda = \frac{vE}{(1+\nu)(1-2\nu)}; \qquad G = \frac{E}{2(1+\nu)}; \quad \text{with} \quad k^2 = \left(\frac{\omega^2\rho}{\lambda + 2G}\right)$$

and a is the radius of the sphere, ρ is the density. When v is equal 0.25 to 0.25, which makes λ equal to C, Eq. (E1) reduces to

$$\frac{\tan(ka)}{ka} = \frac{1}{1 - \frac{3}{4}k^2a^2}$$

Find the first natural frequency of vibration (i.e. value of ω) of a sphere with the following data: $a = 10$ in, $\nu = 0.25$, $\rho g = 0.28$ lb/in^3, and $E = 30 \times 10^6$ psi.

3. Write a Matlab, C or Fortran program to implement the *incremental search method* for finding the root of the equation, $f(x) = 0$, using x_1 as the starting point and $\Delta x^{(1)}$ as the initial step length. Use this program to find the root of the equation

$$f(x) = \frac{1.5x}{(1+x^2)^2} - 0.65 \tan^{-1}\left(\frac{1}{x}\right) + \frac{0.65x}{1+x^2} = 0$$

using $x_1 = 0.0$ and $\Delta x^{(1)} = 0.1$.

4. The van der Waals equation

$$p = \frac{KT}{v-b} - \frac{a}{v^2}$$

relates the specific volume of a gas, v, as a function of its pressure, p, and temperature, T, with K representing the gas constant, whilst a and b are constants.

Using the graphical procedure, find the specific volume of methane gas at a pressure of 100 atm and temperature 0 °F with $a = 4787.4$ ft^4-lbf/lbm^2, $b = 0.0496$ ft^3/lbm, and $K = 96.35$ ft-lbf/lbm-°R, where units lbf and lbm are pounds force and pounds mass, respectively.

Tip: Use 1 atm $= 2116.2$ lbf/ft^2 and $0°$ F $= 459.67°$ R in your calculation.