SKMM 3023 Applied Numerical Methods

Engineering Problem Solving

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Outline

Cognition

- It has to do with how a person understands and acts in the world.
- It is a set of abilities, skills or processes that are part of nearly every human action.
- A process by which the sensory input is transformed, reduced, elaborated, stored, recovered, and used.
- In **science**, cognition is the mental processing that includes the attention of working memory, comprehending and producing language, calculating, reasoning, problem solving, and decision making.
- In psychology and **cognitive science**, "cognition" usually refers to an information processing view of an individual's psychological functions.

Cognitive Process

- It is "the process of thinking".
- **Basic** cognitive process involves obtaining and storing knowledge.
- **Higher** cognitive process presupposes the availability of knowledge and put it to use.

1. Remembering	2. Understanding	3. Applying	4. Analyzing	5. Evaluating	6. Creating

Figure 1: Cognitive Process Dimension.

1. **Remembering:** can the student recall or remember the information?
   keywords: define, duplicate, list, memorize, recall, repeat, reproduce state
2. **Understanding:** can the student explain ideas or concepts?
   keywords: classify, describe, discuss, explain, identify, locate, recognize, report, select, translate, paraphrase
3. **Applying:** can the student use the information in a new way?
   keywords: choose, demonstrate, dramatize, employ, illustrate, interpret, operate, schedule, sketch, solve, use, write.
4. **Analyzing:** can the student distinguish between the different parts?
   keywords: appraise, compare, contrast, criticize, differentiate, discriminate, distinguish, examine, experiment, question, test.
5. **Evaluating:** can the student justify a stand or decision?
   keywords: appraise, argue, defend, judge, select, support, value, evaluate
6. **Creating:** can the student create new product or point of view?
   keywords: assemble, construct, create, design, develop, formulate, write

1. Factual
2. Conceptual
3. Procedural
4. Metacognitive

1. **Factual Knowledge** The basic elements students must know to be acquainted with a discipline or solve problems in it
2. **Conceptual Knowledge** The inter-relationships among the basic elements within a larger structure that enable them to function together
3. **Procedural Knowledge** How to do something, methods of inquiry, and criteria for using skills, algorithms, techniques and methods
4. **Metacognitive Knowledge** Knowledge of cognition in general as well as awareness and knowledge of one's own cognition

Laying the **cognitive process dimension** horizontally, and the **knowledge dimension** vertically, we get a **learning matrix**.

Knowledge Dimension	**Cognitive Process Dimension**					
	1. Remembering	2. Understanding	3. Applying	4. Analyzing	5. Evaluating	6. Creating
1. Factual						
2. Conceptual						
3. Procedural						
4. Metacognitive						

Every engineer should strive to reach some level of metacognitive knowledge and master higher cognitive processes, viz. evaluating & creating.

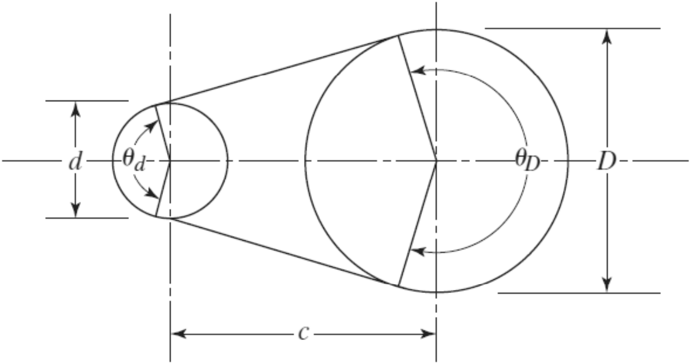


Figure 2: Open belt drive.

The length L of a belt in an open-belt drive, Figure 2, is given by

$$L = \sqrt{4c^2 - (D-d)^2} + \frac{1}{2}\left(D\theta_D + d\theta_d\right) \quad (1)$$

where

$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D-d}{2c}\right) \qquad \theta_d = \pi - 2\sin^{-1}\left(\frac{D-d}{2c}\right)$$

c is the centre distance, D is the diameter of the larger pulley, d is the diameter of the smaller pulley, θ_D is the angle of contact of the belt with the larger pulley, and θ_d is the angle of contact of the belt with the smaller pulley.

If a belt having a length 11 m is used to connect the two pulleys with diameters 0.4 m and 0.2 m, determine the centre distance between the pulleys.

1. **Problem Statement:** Recognise and understand the problem (what is it that needed to be solved?).
2. **Governing Equations or Mathematical Models:** Identify parameters affecting the problem, make the necessary assumptions, develop mathematical model or governing equations (based on theories from Engineering Mathematics and other Engineering Subjects).
3. **Solution:** Solution of the governing equations may make use of the computer programming (why?).
4. **Verification:** Verify and interpret the solution (right/wrong?).

The length of a belt in an open-belt drive, L, is given by

$$L = \sqrt{4c^2 - (D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d) \tag{2}$$

where

$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D-d}{2c}\right) \qquad \theta_d = \pi - 2\sin^{-1}\left(\frac{D-d}{2c}\right)$$

c is the centre distance, D is the diameter of the larger pulley, d is the diameter of the smaller pulley, θ_D is the angle of contact of the belt with the larger pulley, and θ_d is the angle of contact of the belt with the smaller pulley, see Figure-2.8 of Rao (2002).
If a belt having a length 11 m is used to connect the two pulleys with diameters 0.4 m and 0.2 m, determine the centre distance between the pulleys.

- Defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms.
- Its simplest form can be represented as a functional relationship thus

$$\text{Dependent variable} = f(\text{independent variables, parameters, forcing functions})$$

  where
  - **dependent variable:** a characteristic that reflects the behaviour/state of system
  - **independent variables:** dimensions (time, space, mass) along which the system's behaviour that is being determined
  - **parameters:** reflective of system's properties or composition
  - **forcing functions:** external influences acting on the system
- Mathematical model ranges from a simple algebraic relationship to large complicated set of DE. Mathematical models (a.k.a. governing equations) are derived by applying physical laws such as
  - Equilibrium Equation
  - Newton's Law of Motion
  - Conservation Laws: Mass, Momentum, Energy
  - Equation of State

- Specific to our open belt drive problem in Figure 2,

Mathematical Model

$$L = \sqrt{4c^2 - (D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d)$$

where

$$\theta_D = \pi + 2\sin^{-1}\left(\frac{D-d}{2c}\right)$$

$$\theta_d = \pi - 2\sin^{-1}\left(\frac{D-d}{2c}\right)$$

which is a well known relationship, readily derived for us.

- In the majority of engineering problems, the engineer might have to derive the mathematical model from the first principles.

- Solution of the governing equation or mathematical model may appear as
  - Transcendental Functions
  - Linear or Nonlinear Algebraic Equations
  - Homogeneous Equations leading to an Eigenvalue Problem
  - Ordinary or Partial Differential Equations
  - Equations involving Integrals or Derivatives

  which are either closed-form or open-ended.
- Closed-form mathematical expression, e.g.

$$I_1 = \int_a^b x e^{-x^2} dx = \left[-\tfrac{1}{2} e^{-x^2} \right]_a^b = -\tfrac{1}{2} e^{-b^2} + \tfrac{1}{2} e^{-a^2} = \tfrac{1}{2} \left(e^{-a^2} - e^{-b^2} \right)$$

  leads to *analytical solution*
- Open-ended mathematical expressions, e.g.

$$I_1 = \int_a^b f(x) dx = \int_a^b e^{-x^2} dx$$

  need to be *approximated numerically*

- Nowadays, approximated numerical solutions are done by developing a computer program.
- Because numerical methods deal extensively with approximations connected with the manipulation of numbers, accuracy, precision and error feature prominently in programming the solution. We shall cover these later!
- Steps in computer program development:
  - **Algorithm Design:** Listing down of the sequence of steps to define the problem at hand. Techniques available: algorithm, flowchart, pseudocode
  - **Program Coding:** Writing these steps in a computer language.
  - **Debugging:** Testing the program to ensure that it is error-free and reliable.
  - **Documentation:** Making the program easy to understand and use through manual or guide.

Note:

See **SKMM 1013 Programming for Engineers** for details.

- Algorithm: A general sequence of the logical steps in solving a specific problem.
- Flowchart: A graphical representation of the algorithm. Better suited for visualizing complex algorithms.
- Pseudocode: Uses code-like statements in place of the graphical symbols of flowchart. Easier to develop a program with it than with a flowchart.
- Elements of good algorithm
  - Each step must be deterministic i.e. not ambiguous.
  - The process must end after a finite number of steps.
  - The algorithm must be general enough to deal with any contingency.

Solution: Computer Program–Flowchart

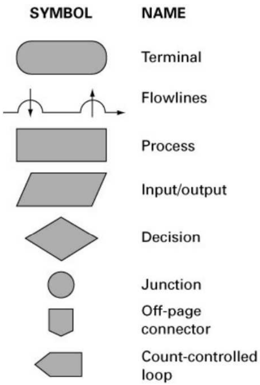


Figure 3: Some of the symbols used in flowcharting.

Name	Function
Terminal	Represents the beginning or end of a program.
Flowlines	Represents the flow of logic. The humps on the horizontal arrow indicate that it passes over and does not connect with the vertical flowlines.
Process	Represents calculations or data manipulations.
Input/Output	Represents inputs or outputs of data and information.
Decision	Represents a comparison, question, or decision that determines alternative paths to be followed.
Junction	Represents the confluence of flowlines.
Off-page Connector	Represents a break that is continued on another page.
Count-controlled loop	Used for loops which repeat a pre-specified number iterations.

Problem Statement:
Find roots of equation $ax^2 + bx + c = 0$ using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Before the actual program is written, we need to outline an algorithm and/or pseudocode for solving this problem:

Algorithm

1. Start
2. Read coefficients a, b and c
3. Implement quadratic formula. Avoid division by zero, allow for complex roots.
4. Display solution i.e. values of x
5. Stop

Pseudocode

```
DO
    READ a, b, c
    root1 = (-b + SQRT(b^2 - 4ac)/(2a)
    root2 = (-b - SQRT(b^2 - 4ac)/(2a)
    PRINT root1, root2
    PRINT 'Try again? Answer yes or no'
    READ response
    IF response = 'no' EXIT
ENDDO
```

- A program is a sequence of instructions to the computer for it to solve a particular problem. A set of programs is called code.
- Programs are written in some programming language, e.g. C/C++, Fortran, Matlab, Basic, Pascal, Java.
- Programs are stored in files which are a sequence of bytes which is given a name and stored on a disk.
- A program is a file containing a sequence of statements, each of which tells the computer to do a specific action.
- Once a program is run or executed the commands are followed and actions occur in a sequential manner.
- If the program is designed to interact with the outside world, then it must have input and output.
- A program is said to have a bug if it contains a mistake or it does not function in the way it is intended to.
- Bugs can happen both in the logic of the program, and in the commands.

The final step of any engineering analysis should be the verification of results.

- Various sources of error can contribute to wrong results. Common sources of error include:
  - misunderstanding a given problem,
  - making incorrect assumptions to simplify the problem,
  - applying a physical law that does not truly fit the given problem, and
  - incorporating inappropriate physical properties
- Before you present your solution or the results to your instructor or, later in your career, to your manager, you need to learn to think about the calculated results. You need to ask yourself the following question:
  - Do the results make sense?

  A good engineer must always find ways to check results.
- Ask yourself this additional question:
  - What if I change one of the given parameters. How would that change the result?

  Then consider if the outcome seems reasonable.

- If you formulate the problem such that the final result is left in parametric (symbolic) form, then you can experiment by substituting different values for various parameters and look at the final result.
- In some engineering work, actual physical experiments must be carried out to verify one's findings.
- Starting today, get into the habit of asking yourself if your solution to a problem makes sense.
- Asking your instructor if you have come up with the right answer or checking the back of your textbook to match answers are not good approaches in the long run. You need to develop the means to check your results by asking yourself the appropriate questions.
- Remember, once you start working for hire, there are no answer books. You will not want to run to your boss to ask if you did the problem right!

Problem Statement:

Assuming that the thrust T of a screw propeller is dependent upon diameter D, speed of advance v, fluid density ρ, rotational speed of propeller N and coefficient of viscosity μ, derive and expression that relates all the parameters involved and solve for T.

Mathematical Model:

Through *dimensional analysis*

$$T = \rho v^2 D^2 f\left(\frac{\mu}{\rho v D}, \frac{ND}{v}\right)$$

Problem Statement:
Given temperature in degrees Fahrenheit, the temperature in degrees Kelvin is to be computed and shown.

Mathematical Model:
From Physics, these two temperature scales are related through

$$T_k = \left(\frac{T_F - 32}{1.8}\right) + 273.15$$

and the parameters involved in this problem are T_K and T_F

Algorithm

1. Start
2. Get the temperature in Fahrenheit, T_F
3. Compute the temperature in Kelvin using the formula:

$$T_k = \left(\frac{T_F - 32}{1.8}\right) + 273.15$$

4. Show the temperature in Kelvin, T_k
5. Stop

Pseudocode

```
Start
Read TF
TK = (TF-32)/1.8  + 273.15
Print TK
Stop
```

Problem Statement:

A bungee jumper with a mass of 68.1 kg leaps from a stationary hot air balloon. Compute velocity for the first 12 s of free fall and determine the terminal velocity that will be attained for an infinitely long cord. Use a drag coefficient of 0.25 kg/m and the acceleration due to gravity is 9.81 m/s.

Mathematical Model:

The downward force, F_D, and upward force, F_U, are, respectively,

$$F_D = mg \qquad \text{and} \qquad F_U = -c_d v^2$$

The net force, F, on the jumper is the difference between F_D and F_U. Therefore,

$$F = F_D + F_U \quad \Longrightarrow \quad ma = mg - c_d v^2 \quad \Longrightarrow \quad m\frac{dv}{dt} = mg - c_d v^2$$

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2 \qquad \text{(E0)}$$

Solution:
If the jumper is initially at rest ($v = 0$ at $t = 0$), calculus can be used to solve Eq. (E0) for

$$v(t) = \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}} t\right) \tag{E1}$$

Algorithm

1. Start
2. Assign values to parameters and constant (g, m, c_d)
3. Create vector containing $0 < t < 20$, in steps of 2
4. Evaluate Eq. (E1), where v is computed for each value of t, and the result is assigned to a corresponding position in the v array
5. Display solution by plotting the graph of v vs. t
6. Stop

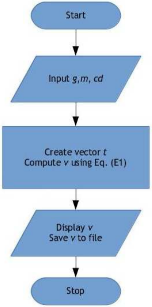
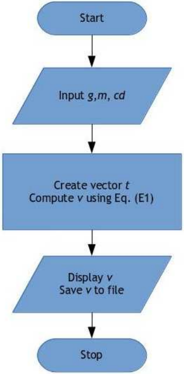


Figure 4: Flowchart.

Solution:



Matlab Code

```
% Assign values to the parameters:
g = 9.81; m = 68.1; cd = 0.25;

% Create column vector that contains values
% 0 < t < 20 in steps of 2:
t = [0:2:20]';

% Evaluate Eq. (E1) where the formula is
% computed for each value of the t array, and
% the result id assigned to a corresponding
% position in the v array:
v = sqrt(g*m/cd) * tanh(sqrt(g*cd/m)*t);

% Plot graph of velocity (v) versus time (t):
plot(t, v)
title('Plot of v versus t')
xlabel('Time t (sec)'); ylabel('Velocity v (m/s)');
grid on
```

Accuracy and Precision

Because numerical methods deal extensively with approximations connected with the manipulation of numbers, accuracy, precision and error feature prominently in programming the solution. We shall now look at them in more details.

Errors associated with calculations and measurements can be characterized with regard to their accuracy and precision.

- **Accuracy** refers to how closely a computed or measured value agrees with true value. The opposite, *inaccuracy* (also called *bias*), is defined as systematic deviation from truth.
- **Precision** refers to how closely individual computed or measured value agrees with each other. The opposite, *imprecision* (also called *uncertainty*), refers to the magnitude of the scatter.

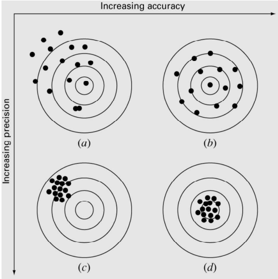


Figure 5: Concepts of accuracy and precision.(a) Inaccurate and imprecise; (b) accurate and imprecise; (c) inaccurate and precise; (d) accurate and precise.

Accuracy and Precision

- **Implied Precision** When writing down a measurement as a decimal number, there is an implied level of precision, namely, 0.5 unit in the last position. For example, a measurement of 23.534 implies that the maximum error is correct to at least 0.0005.
  Alternatively, it may be convenient to write down a measurement with the maximum error explicitly given: 23.534 ± 0.012, implying that the actual answer lies in the interval (23.522, 23.546). While such a notation is useful for the actual study of error propagation, this will not be used much in this course.
  Implied precision is a measure of absolute error, covered later!
- Numerical methods should be
  - sufficiently accurate or unbiased to meet the requirements of a particular engineering problem,
  - precise enough for adequate engineering design.

Error

- **Error** is the collective term to represent both inaccuracy and imprecision of predictions by numerical methods. If $\tilde{x}$ is an approximation of true value, x, then ...
  - true or absolute error is defined as

$$E_x = x - \tilde{x} \tag{3}$$

  - and relative error is defined as

$$R_x = \left| \frac{x - \tilde{x}}{x} \right|, x \neq 0 \tag{4}$$

- $\tilde{x}$ is an approximation of x to d significant digits if d is the largest integer for which

$$\left| \frac{x - \tilde{x}}{x} \right| < \frac{1}{2} 10^{-d} \tag{5}$$

Problem Statement:
Suppose that you are asked to measure the lengths of a bridge and a rivet, and came up with 9,999 cm and 9 cm, respectively. If the true values are 10,000 cm and 10 cm, respectively, compute the *absolute* error and the *relative* error (in %) for each case.

Solution:
Absolute error for measuring

$$\begin{aligned}\text{bridge:}\quad & E_x = x - \tilde{x} = 10000 - 9999 = 1\ \text{cm}\\ \text{rivet:}\quad & E_x = x - \tilde{x} = 10 - 9 = 1\ \text{cm}\end{aligned}$$

Percent relative error for measuring

$$\begin{aligned}\text{bridge:}\quad & R_x = \left|\frac{x - \tilde{x}}{x}\right| \times 100 = \left|\frac{1}{10000}\right| \times 100 = 0.01\%\\ \text{rivet:}\quad & R_x = \left|\frac{x - \tilde{x}}{x}\right| \times 100 = \left|\frac{1}{10}\right| \times 100 = 10\%\end{aligned}$$

Problem Statement:

1. What are the absolute and relative errors of the approximation 3.14 to the value π?
2. A resistor labeled as 240 Ω is actually 243.32753 Ω. What are the absolute and relative errors of the labeled value?

Solution:

1. Errors

$$\text{Absolute:} \quad E_x = x - \tilde{x} = \pi - 3.14 \approx 0.0016$$

$$\text{Relative:} \quad R_x = \left|\frac{x - \tilde{x}}{x}\right| = \left|\frac{\pi - 3.14}{\pi}\right| \approx 0.00051$$

2. Errors

$$\text{Absolute:} \quad E_x = x - \tilde{x} = 243.32753 - 240 \approx 3.3\ \Omega$$

$$\text{Relative:} \quad R_x = \left|\frac{x - \tilde{x}}{x}\right| = \left|\frac{243.32753 - 240}{243.32753}\right| \approx 0.014$$

Problem Statement:

Given $x = e$ (where e is a constant and base of the natural logarithm $= 2.718281828$) is approximated by $\tilde{x} = 2.71828$, find

- absolute and relative errors,
- number of significant digits d to which $\tilde{x}$ approximates x.

$$\left| \frac{x - \tilde{x}}{x} \right| < \frac{1}{2} \times 10^{-d}$$

Solution:

Work through the example.

- How do we determine error estimates in the absence of knowledge regarding the true value?
- Example: Many numerical methods use an *iterative approach* to compute answers. In such approach, a present approximation is made on the basis of a previous approximation i.e. process is performed repeatedly, or iteratively, to successfully compute better and better approximations.
- In this case, error is estimated as the difference between previous and current approximations, thus

$$\epsilon = \frac{\text{current approximation} - \text{previous approximation}}{\text{current approximation}}$$

1. Errors in mathematical modeling:
   - simplifying approximation,
   - assumption made in representing physical system by mathematical equations
2. Blunders:
   - undetected programming errors,
   - silly mistakes
3. Errors in input:
   - due to unavoidable reasons e.g. errors in data transfer,
   - uncertainties associated with measurements
4. Machine errors:
   - rounding,
   - chopping,
   - overflow,
   - underflow
5. Truncation errors associated with mathematical process:
   - approximate evaluation of an infinite series,
   - integral involving infinity

- Number is expressed as fractional part, called a mantissa or significand and an integer part, called an exponent or characteristic

  $$m \cdot b^e$$

  where m is mantissa, b is the base of the number system being used and e the exponent. If the number has leading zeros digits, the mantissa is usually normalized. If $1/34 = 0.029411765\ldots$ were to be stored in a floating-point base-10 system that allows only four decimal places to be stored, then $1/34$ would be stored as

  $$1/34 = 0.0294 \times 10^0 \rightarrow 0.2941 \times 10^{-1}$$

- The discrepency introduced by the use of an approximate expression in place of an exact mathematical expression.
- Example: Taylor's series expansion of $\ln(1+x)$

$$y(x) = \ln(1+x) = \sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i} x^i$$
$$= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5 - \frac{1}{6}x^6 + \frac{1}{7}x^7 + \ldots; \quad |x| \leq 1$$

If $y(x)$ is approximated by the first four terms of this Taylor's series, the resulting discrepency between the exact function $y(x)$ and the approximate function $\tilde{y}(x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$, is called the trunction error.

- Computer can only store a finite number of digits, so actual numbers may undergo chopping or rounding.
- Let a decimal number $x = 0.b_1 b_2 \dots b_i b_{i+1} b_{i+2}$ where $0 \leq b_i \leq 9$ for $i \geq 1$. If the maximum number of decimal digits used in the floating-point computation is i:
  - chopped floating-point representation of x is $x_{\text{chop}} = 0.b_1 b_2 \dots b_i$ where ith digit of x_{chop} is identical to the ith digit of x.
  - rounded floating-point representation of x is $x_{\text{round}} = 0.b_1 b_2 \dots b_{i-1} d_i$where $d_i (1 \leq d_i \leq 9)$ is obtained by rounding the number $d_i d_{i+1} d_{i+2} \dots$ to the nearest integer.
- Numerical solution of engineering problem uses suitable algorithm and local computational errors involved in various steps of this algorithm will accumulate to a computational error in output
- Local computational error arise due to errors involved during arithmetic operations such as
  - subtraction of numbers of near-equal magnitude,
  - irrational numbers (such as $\sqrt{3}$ and π) being replaced by machine numbers with finite number of digits

Problem Statement:
The value of e is given by $e = 2.718281828459045\ldots$. Show the seven-digit representations of e by chopping and rounding are

$$e_{\text{chop}} = 0.2718281 \times 10^1 \qquad e_{\text{round}} = 0.2718282 \times 10^1$$

Solution:
Work through the example.

- Error in the output of a procedure due to the error in the input date
- Output of a procedure f is a function of input parameters $(x_1, x_2, \ldots, x_n)$

$$f = f(x_1, x_2, \ldots, x_n) \equiv f(\vec{X})$$

- Value of f is found by Taylor's series expansion about the approximate values $\vec{\bar{X}} = \{\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n\}^T$ as

$$\begin{aligned} f(x_1, x_2, \ldots, x_n) = f(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) &+ \frac{\partial f}{\partial x_1}(\vec{\bar{X}})(x_1 - \bar{x}_1) \\ &+ \frac{\partial f}{\partial x_2}(\vec{\bar{X}})(x_2 - \bar{x}_2) + \ldots + \frac{\partial f}{\partial x_n}(\vec{\bar{X}})(x_n - \bar{x}_n) \\ &+ \text{higher order derivative terms} \end{aligned}$$

Propagation of Error

- Neglecting higher order derivative terms, the error in the output can be expressed as

$$\Delta f = f - \bar{f}$$
$$\equiv f(x_1, x_2, \ldots, x_n) - f(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$$

and denoting errors in input parameters as

$$\Delta x_i = x_i - \bar{x}_i, \quad i = 1, 2, \ldots, n$$

we can estimate propagation error as

$$\Delta f \approx \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(\vec{\bar{X}})(x_i - \bar{x}_i)$$

Taylor's series expansion of $f(x)$ in Figure 6 at a known point x_i for a given step size h yields

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \frac{f'''(x_i)}{3!}h^3 + \ldots$$

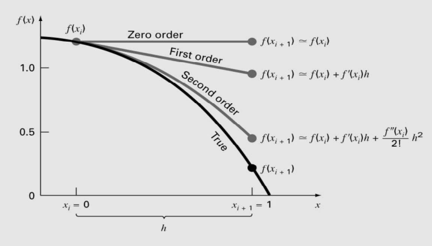


Figure 6: Relative margin of error for neglecting higher order terms of Taylor's series.

Propagation of Error

- If $f(x_1, x_2, \ldots, x_n) \neq 0$ and $x_i \neq 0$, the relative propagation error, ε_f, is

$$\varepsilon_f = \frac{\Delta f}{f} = \sum_{i=1}^{n} \left\{ \frac{x_i}{f(\vec{X})} \frac{\partial f}{\partial x_i}(\vec{\bar{X}}) \right\} \varepsilon_{x_i} \tag{6}$$

where ε_{x_i} is relative error in x_i

$$\varepsilon_{x_i} = \frac{x_i - \bar{x}_i}{x_i} \quad i = 1, 2, \ldots, n$$

- The quantity

$$c_i = \left\{ \frac{x_i}{f(\vec{X})} \frac{\partial f}{\partial x_i}(\vec{\bar{X}}) \right\}$$

in Eq. (6) is called the amplification or condition number of relative input error ε_{x_i}.

- When two numbers are used in an arithmetic operations, the numbers cannot be stored exactly by the floating-point representation.
- Let x and y be the exact number and $\tilde{x}$ and $\tilde{y}$ their approximate values. Then

$$x = \tilde{x} + \varepsilon_x \qquad\qquad y = \tilde{y} + \varepsilon_y$$

  ε_x and ε_y denote errors in x and y, respectively.
- When arithmetic operation, say multiplication, is carried out on the numbers, associated error, E, results

$$E = xy - \tilde{x}\tilde{y} = xy - (x - \varepsilon_x)(y - \varepsilon_y) = x\varepsilon_y + y\varepsilon_x - \varepsilon_x\varepsilon_y$$

  and relative error, R, is

$$R = \frac{E}{xy} = \frac{\varepsilon_x}{x} + \frac{\varepsilon_y}{y} - \frac{\varepsilon_x}{x}\frac{\varepsilon_y}{y} = R_x + R_y - R_xR_y \approx R_x + R_y$$

  where $|R_x| << 1$ and $|R_y| << 1$

Problem Statement:
The deflection y of the top of a sailboat mast is

$$y = \frac{FL^4}{8EI}$$

where F is a uniform side loading (lb/ft), L is height (ft), E is the modulus of elasticity (lb/ft^2), and I is the moment of inertia (ft^4). Estimate the error in y given the following data:

$$F = 50 \text{ lb/ft} \qquad L = 30 \text{ ft} \qquad E = 1.5 \times 10^8 \text{ lb/ft}^2 \qquad I = 0.06 \text{ ft}^4$$

$$\Delta F = 2 \text{ lb/ft} \qquad \Delta L = 0.1 \text{ ft} \qquad \Delta E = 0.01 \times 10^8 \text{ lb/ft}^2 \qquad \Delta I = 0.0006 \text{ ft}^4$$

Solution:
Work through the example.

Further Reading

Your homework!

- Read Section 1, pp 1–13 of STEPHEN J. CHAPMAN (2001): *MATLAB Programming for Engineers, 2ed*, Brooks/Cole
- Read part of Chapter 1, pp. 1–40 of RICHARD L. BURDEN & J. DOUGLAS FAIRES (2011): *Numerical Analysis, 9ed*, ISBN-13: 978-0-538-73351-9, Brooks/Cole

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3. David Kincaid & Ward Cheney (1991): *Numerical Analysis: Mathematics of Scientific Computing*, ISBN 0-534-13014-3, Brooks/Cole Publishing Co.
4. Steven C. Chapra (2012): *Applied Numerical Methods with MATLAB for Engineers and Scientists, 3ed*, ISBN 978-0-07-340110-2, McGraw-Hill
5. John H. Mathews & Kurtis D. Fink (2004): *Numerical Methods Using Matlab, 4ed*, ISBN 0-13-065248-2, Prentice Hall
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