Taylor Series

Any smooth function can be approximated by a polynomial. Expanding the function expression about (around) x, where a is near x;

$$f(x) = f(a) + f^{'}(a)(x-a) + \frac{f^{''}(a)}{2!}(x-a)^2 + \frac{f^{'''}(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n$$

where R_n is the *remainder* and defined (in *integral* form)

$$R_n = \int_a^x \frac{(x-t)^n}{n!} f^{(n+1)}(t)\mathrm{dt}$$

where $t =$ a dummy variable

In *derivative* form, the remainder is expressed as

$$R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-a)^{n+1}$$

where ξ lies between x and a